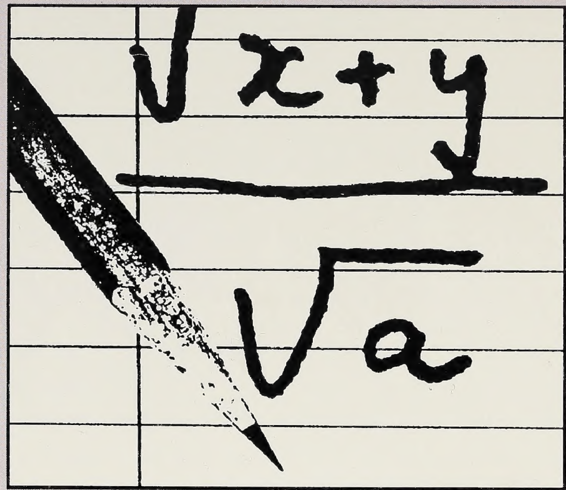
MATHEMATICS 30

Powers and Radicals



Unit 1

Distance Learning

Alberta EDUCATION

Welcome

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

Mathematics 33 Student Module Unit 1 Powers and Radicals Alberta Distance Learning Centre ISBN No. 0-7741-0104-0

General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.

- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.

- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.

- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.

- **Extensions** gives you the opportunity to take the topic one step further.

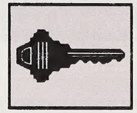
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.

- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

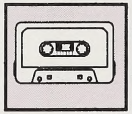
Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

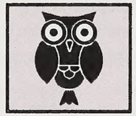
An explanation of what they mean is written beside each visual cue.



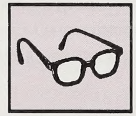
Key Idea
- flagging important ideas



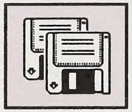
Audiotape
- learning by listening to an audiotape



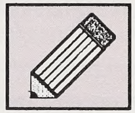
What You Already Know
- reviewing what you already know



Another View
- exploring different perspectives

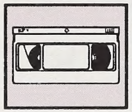


Computer Software
- learning by using computer software



Review
- studying previous concepts

Solutions
- correcting the activities

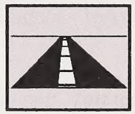


Videotape
- learning by viewing a videotape

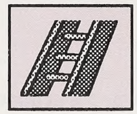
Introduction
- introducing the unit

Extra Help
- providing additional study

Print Pathway
- choosing a print alternative



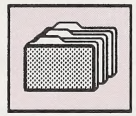
What Lies Ahead
- previewing the unit



Extensions
- going on with the topic

Calculator
- using your calculator

Exploring the Topic
- actively learning new concepts



What You Have Learned
- summarizing what you have learned

Mathematics 33

Course Overview

Mathematics 33 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Powers and Radicals	10%
Unit 2 Polynomials and Rational Expressions	10%
Unit 3 Functions and Relations	16%
Unit 4 Quadratic Functions and Equations	20%
Unit 5 Trigonometry	16%
Unit 6 Statistics	16%
Unit 7 Annuities	6%
Unit 8 Mortgages and Loans	6%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Powers and Radicals

This unit covers topics dealing with powers and radicals. Each topic contains explanations, examples, and activities to assist you in understanding powers and radicals. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in the **Appendix**. In several cases there is more than one way to do the question.

Unit 1 Powers and Radicals

Contents at a Glance

Powers and Radicals

Aviation technologists are one group of people who frequently perform calculations involving rational exponents and radicals. Such calculations are used when aircraft components are tested in wind tunnels. You have worked previously with exponents and radicals and should have some knowledge of the relationships between the two.

To this point you were concerned with the symbols involved. You will now extend your work to simplifying calculations and solving equations using radicals. Being able to work in these areas will be beneficial when applied in the field of technology, ranging from effectively powering a wrecking ball to the analysis and testing of instruments used in the aviation industry.

Unit 1 Powers and Radicals

Topic 1
Changing the Form of a Radical and Adding and Subtracting Radicals

Topic 2
Multiplying and Dividing Radicals

Topic 3
Solving and Applying Radical Equations

What You Already Know

Mathematical ideas are often linked in pairs of opposites. Mathematical opposites take you from a starting point to a new point and, in reverse, take you from a new point to the starting point. Exponents and radicals are one such pair of opposites. Before you begin this unit, review the following skills.

- The square of 5 is 25 or $5^2 = 25$, while the square root of 25 is 5 or $\sqrt{25} = 5$.

- Similarly, $4^3 = 64$, while $\sqrt[3]{64} = 4$.

- In an exponential expression such as 7^2, the 7 is the base and the 2 is the exponent. The expression 7^2 is called a power.

- In a radical expression such as $\sqrt[4]{81}$, the 81 is the radicand, the 4 is the index, and the symbol $\sqrt[4]{}$ is the radical sign.

- The following table will help you recall the laws of exponents.

Law	Example	Symbolic Representation
Product Law	$3^2 \bullet 3^4 = 3^6$	$x^a \bullet x^b = x^{a+b}$
Quotient Law	$4^{10} \div 4^2 = 4^8$	$x^a \div x^b = x^{a-b}$
Power Law	$\left(2^3\right)^4 = 2^{12}$	$\left(x^a\right)^b = x^{ab}$
Power of a Product	$(3 \cdot 5)^2 = 3^2 \bullet 5^2$	$(xy)^a = x^a \bullet y^a$
Power of a Quotient	$\left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5}$	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

- The laws of exponents apply when the exponents are natural numbers $\{1, 2, 3, \ldots\}$, integers $\{\ldots -2, -1, 0, 1, 2, \ldots\}$, or rational numbers (fractions).

- Recall that any base to the zero power is equal to one.
  $x^0 = 1 \quad (x \neq 0)$

- Recall that $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$, where $x \neq 0$.
  It is sometimes easier to think in terms of positive exponents, so negatives are changed to positives.

- $x^{\frac{a}{b}} = \sqrt[b]{x^a}$ or $x^{\frac{a}{b}} = \left(\sqrt[b]{x}\right)^a$

$$4^{\frac{3}{2}} = \sqrt{4^3} \quad \text{or} \quad 4^{\frac{3}{2}} = \left(\sqrt{4}\right)^3$$
$$= \sqrt{64} \qquad\qquad = 2^3$$
$$= 8 \qquad\qquad = 8$$

- When the index of a radical is not specified, it is understood to be two.

$$\sqrt{100} = \sqrt[2]{100}$$
$$= 10$$

- Like fractions have the same denominators. Only like fractions can be added or subtracted.

$$\frac{3}{4} + \frac{7}{10} = \frac{15}{20} + \frac{14}{20} \qquad \frac{3}{4} - \frac{7}{10} = \frac{15}{20} - \frac{14}{20}$$
$$= \frac{29}{20} \qquad\qquad = \frac{1}{20}$$
$$= 1\frac{9}{20}$$

- Like radicals are expressions which have the same index and the same radicand.

The expressions $3\sqrt{2}, \frac{1}{2}\sqrt{2}$, and $3a\sqrt{2}$, are like radicals because they all contain the radical $\sqrt{2}$.

The expressions $16\sqrt[5]{4}, -4\sqrt[5]{17}$, and $-7x\sqrt[5]{-9}$ are not like radicals because the radicands are not the same.

Now that you have reviewed some important basic concepts related to this unit, do the following **Review**.

Review

Do the following questions to confirm your understanding of the concepts mentioned previously.

1. Evaluate.

   a. 6^3 b. 5^{-2}

   c. $(-2)^4$ d. 7^0

2. Simplify.

   a. $t^4 \times t^5$ b. $k^8 \div k^3$

   c. $\left(a^4\right)^3$ d. $\left(\frac{p^3}{q}\right)^2$

   e. $\left(m^3 n\right)^2$ f. $\left(p^5\right)^{-2}$

   g. $t^{-7} \times t^5$ h. $m^{-1} \div m^{-3}$

   i. $\dfrac{\left(m^{-3} n^{-5}\right)\left(m^2 n^4\right)}{\left(m^2 n\right)^{-2}}$

3. Express in exponential form.

a. $\sqrt[3]{4}$

b. $\left(\sqrt{a}\right)^3$

c. $\dfrac{1}{\left(\sqrt[3]{y}\right)^2}$

4. Express in radical form.

a. $(2)^{\frac{1}{2}}$

b. $h^{\frac{2}{3}}$

c. $x^{-\frac{1}{3}}$

d. $d^{-\frac{2}{5}}$

5. Evaluate.

a. $\sqrt[3]{-27}$

b. $\left(\sqrt{25}\right)^3$

c. $\sqrt[3]{(-1)}^4$

d. $\left(\sqrt{9}\right)^0$

e. $49^{\frac{3}{2}}$

f. $(-64)^{\frac{1}{3}}$

g. $25^{-\frac{1}{2}}$

h. $(-8)^{\frac{2}{3}}$

6. Find the sums or differences.

a. $\frac{5}{6}+\frac{7}{18}$

b. $1\frac{1}{3}+7\frac{1}{2}+4\frac{2}{9}$

c. $\frac{11}{12}-\frac{4}{9}$

d. $5\frac{1}{3}-2\frac{7}{10}$

7. Classify each group of expressions as like radicals or unlike radicals.

a. $3\sqrt{3}$ $15\sqrt{3}$ $3c\sqrt{3}$

b. $16\sqrt{5}$ $3\sqrt[5]{17}$ $9\sqrt{10}$

c. $7\sqrt[3]{3x}$ $6.2\sqrt[3]{3x}$ $\frac{3}{4}\sqrt[3]{3x}$

If you had difficulties with this review, you may need to look back at **Math 23, Unit 1, Powers and Radicals**.

Now go to the **Review** solutions in the **Appendix**.

Topic 1 Changing the Form of a Radical and Adding and Subtracting Radicals

Introduction

When participating in games such as hockey, baseball, soccer, and basketball, there are certain rules and objectives which must be followed.

Similarly, you must know the goals and guidelines when working with radicals. In order to perfom arithmetic operations with radicals, you must be able to identify the simplest form for a radical. Also, you must be able to change a radical to its equivalent simplest form. Your knowledge of perfect squares should strengthen and expand your existing skills when working with radicals.

The symbols used for radicals are universal. These symbols were not always as they are today. Over the centuries the present-day symbols seemed to work and now are accepted worldwide in all languages.

What Lies Ahead

Throughout the topic you will learn to

1. change entire radicals to mixed radicals
2. change mixed radicals to entire radicals
3. add and subtract radicals

Now that you know what to expect, turn the page to begin your study of changing the form of a radical and adding and subtracting radicals.

Exploring Topic 1

Activity 1

Change entire radicals to mixed radicals.

Radicals

Often it is important or necessary to find the square root of a number. By now you know that an exact square root cannot be found for most numbers. The only numbers for which an exact square root can be found are perfect square numbers such as one, four, nine, sixteen, and twenty-five. It may also be necessary to find cube roots, fourth roots, fifth roots, and many others.

The use of a radical symbol such as $\sqrt{x}$ allows mathematics to remain very precise. Note the following examples:

- In $\sqrt{11} \doteq 3.317$, $\sqrt{11}$ is an exact value while 3.317 is an approximate value to the nearest thousandth.
- In $\sqrt[5]{62} \doteq 2.283$, $\sqrt[5]{62}$ is an exact value while 2.283 is an approximate value to the nearest thousandth.

In the study of mathematics and science there are many formulas which involve calculations using radicals.

If a deep-sea diver is submerged and wants to find the velocity of a wavelength, the formula $v = \sqrt{\dfrac{gh}{2\pi}}$ would be used, where v is in metres per second, h is in metres, and g is acceleration due to gravity.

To find c, the hypotenuse of a right-angled triangle, you would use $c = \sqrt{a^2 + b^2}$.

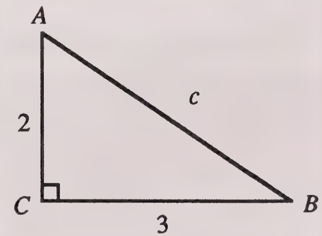
As you continue your study of radical expressions and when you learn to simplify radicals, you must keep an important property in mind. This property is illustrated as follows:

$$\sqrt{36} = \sqrt{9 \times 4} \text{ or } \sqrt{9} \times \sqrt{4}$$
$$\sqrt{200} = \sqrt{100 \times 2} \text{ or } \sqrt{100} \times \sqrt{2}$$
$$\sqrt[3]{240} = \sqrt[3]{8 \times 30} \text{ or } \sqrt[3]{8} \times \sqrt[3]{30}$$

This procedure allows you to simplify radicals.

Perfect square numbers are
1, 4, 9, 16, 25, ...

Perfect cube numbers are
1, 8, 27, 64, 125, ...



Use the Pythagorean theorem to find c:

$$c^2 = a^2 + b^2$$
$$c^2 = 2^2 + 3^2$$
$$c^2 = 4 + 9$$
$$c^2 = 13$$
$$c = \sqrt{13}$$

Use your calculator to calculate $\sqrt{13}$ to the nearest tenth:
$\sqrt{13} \doteq 3.6$

In symbolic language this property is stated as follows:
If $a \geq 0$ and $b \geq 0$, then
$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ and
$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$.

Changing Entire Radicals to Mixed Radicals

All radicals are not of the same type. Radicals can be entire radicals or mixed radicals. An entire radical is an expression which is written with a real number under the radical sign, such as $\sqrt{45}$.

Using the property $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, the entire radical $\sqrt{45}$ can be changed to a mixed radical.

$$\begin{aligned}\sqrt{45} &= \sqrt{9} \times \sqrt{5}\\ &= 3\sqrt{5}\end{aligned}$$

multiplier radical

The 9 is the largest perfect square that divides evenly into 45.

By dividing out the largest possible perfect square number, you are putting the radical into its simplest form.

Example 1

- Express the entire radical $\sqrt{150}$ as a mixed radical in simplest form.

Solution:

$$\begin{aligned}\sqrt{150} &= \sqrt{25 \times 6}\\ &= \sqrt{25} \times \sqrt{6}\\ &= 5\sqrt{6}\end{aligned}$$

- Express the entire radical $\sqrt[3]{56}$ as a mixed radical in simplest form.

Solution:

$$\begin{aligned}\sqrt[3]{56} &= \sqrt[3]{8 \times 7}\\ &= \sqrt[3]{8} \times \sqrt[3]{7}\\ &= 2\sqrt[3]{7}\end{aligned}$$

The simplified form for an entire radical is usually a mixed radical. A radical is in simplest form when its radicand is as small as possible; that is, the radicand has no factor other than 1 which is a perfect square.

Recall some perfect squares.

x	x^2
1	1
2	4
3	9
4	16
5	25
6	36

x	x^2
7	49
8	64
9	81
10	100
11	121
12	144

Note: A mixed radical is a product which has two parts, the multiplier or coefficient and the radical.

multiplier $\rightarrow 3\sqrt{5} \leftarrow$ radical

When simplifying a radical, it is helpful to recognize the highest perfect square term or value under the radical sign.

Do parts a and c of questions 1 to 4. Do all of question 5.
If you need more practice, do part b of questions 1 to 4.

1. Change the following to mixed radicals.

   a. $\sqrt{20}$

   b. $\sqrt{300}$

   c. $\sqrt[3]{72}$

2. Express each product as a mixed radical.

   a. $\sqrt{4} \times \sqrt{3}$

   b. $\sqrt[3]{27} \times \sqrt[3]{9}$

   c. $\sqrt{36} \times \sqrt{5}$

3. Simplify each of the following.

   a. $\sqrt{400}$

   b. $\sqrt{225}$

   c. $\sqrt[3]{625}$

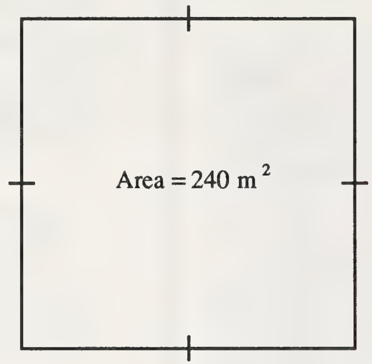
4. Express each of the following as a mixed radical in simplest form.

   a. $\sqrt{48}$ b. $\sqrt{63}$

   c. $\sqrt{1000}$ d. $\sqrt[3]{2000}$

5. a. Express the length of each side of the following square as a mixed radical.

   b. Use your calculator to express your answer to the nearest tenth of a metre.



For solutions to **Activity 1**, turn to the **Appendix, Topic 1**.

Activity 2

Change mixed radicals to entire radicals.

To change a mixed radical to an entire radical, reverse the property $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ and use $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$. Substituting actual numbers for a and b, you get, for example, $\sqrt{8} = \sqrt{4} \times \sqrt{2}$ which is $2\sqrt{2}$ and $\sqrt{4} \times \sqrt{2} = \sqrt{8}$, respectively. Study the following example to clarify the changing process.

Example 2

- Express $4\sqrt{10}$ as an entire radical.

Solution:

$$\begin{aligned} 4\sqrt{10} &= \sqrt{4^2} \times \sqrt{10} \\ &= \sqrt{16} \times \sqrt{10} \\ &= \sqrt{16 \times 10} \\ &= \sqrt{160} \end{aligned}$$

- Express $7\sqrt[3]{4}$ as an entire radical.

Solution:

$$\begin{aligned} 7\sqrt[3]{4} &= \sqrt[3]{7^3} \times \sqrt[3]{4} \\ &= \sqrt[3]{343} \times \sqrt[3]{4} \\ &= \sqrt[3]{343 \times 4} \\ &= \sqrt[3]{1372} \end{aligned}$$

See if you can apply what you have learned by doing the following questions. You may choose not to do all of the problems. You decide how much drill you need.

If you are uncertain about this concept after doing this activity, you may choose to do the **Extra Help** section. Do the **Extensions** section if you wish to be challenged with more difficult problems.

Do at least parts a and c of the following questions.

1. Change the following to entire radicals.

   a. $3\sqrt{5}$

   b. $4\sqrt{3}$

   c. $7\sqrt[3]{2}$

Recall: Do you remember how to change a real number to a radical?

$$\begin{aligned} 3 &= \sqrt{3 \times 3} \\ &= \sqrt{9} \end{aligned} \qquad \begin{aligned} 7 &= \sqrt{7 \times 7} \\ &= \sqrt{49} \end{aligned}$$

Remember to square the multiplier of a mixed radical when putting it under the radical sign.

2. Express each product as an entire radical.

a. $\sqrt{2} \times \sqrt{5}$

b. $\sqrt{6} \times \sqrt{3}$

c. $\sqrt[3]{11} \times \sqrt[3]{7}$

For solutions to **Activity 2**, turn to the **Appendix, Topic 1**.

Activity 3

Add and subtract radicals.

Your knowledge of simplifying radicals is very useful when operations such as addition and subtraction are applied. In previous courses you learned how to add and subtract polynomials. The skills learned to do those operations will be applied to the addition and subtraction of radicals.

Compare the addition of polynomials to the addition of radicals.

$4y$ and $6y$ $\qquad$ $3\sqrt{2}$ and $5\sqrt{2}$

Both pairs are like terms. In the first pair, y makes the two terms like. In the second pair, $\sqrt{2}$ makes the terms like. The sum for both pairs is found by adding the numerical coefficients and repeating the portion which is like or common.

Complete the addition of the previous terms.

$4y + 6y = 10y \qquad 3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$

The terms in the polynomial $5x + 6y$ are unlike terms. The terms in the expression $5\sqrt{3} + 6\sqrt{2}$ are also unlike terms because the radicands are different. (The radicand is the number under the radical sign.)

$$\underbrace{5x + 6y}_{\text{unlike}} \qquad \underbrace{5\sqrt{3} + 6\sqrt{2}}_{\text{unlike}}$$

This is the simplest way in which the sum of unlike terms can be written.

To simplify radical expressions which are mixed radicals, you add or subtract like radicals.

Carefully study the following examples.

Recall: In the expression $5a$, the numerical coefficient is 5 and the literal coefficient is a.

Keep in mind that the same rules that apply to addition also apply to subtraction.

$$\begin{aligned} 8x - 5x &= 8x + (-5x) \\ &= 3x \end{aligned}$$

Recall: Like radicals have the same radicands.

Example 3

Simplify the following radical expressions.

- $\sqrt{3}+2\sqrt{3}$

Solution:

$$\begin{aligned}\sqrt{3}+2\sqrt{3} &= 1\sqrt{3}+2\sqrt{3}\\ &= (1+2)\sqrt{3}\\ &= 3\sqrt{3}\end{aligned}$$

Use the distributive property. If a, b, and c are real numbers, then $ab+ac=a(b+c)$.

- $6\sqrt{5}+9\sqrt{2}+2\sqrt{5}-9\sqrt{2}$

Solution:

$$\begin{aligned}6\sqrt{5}+9\sqrt{2}+2\sqrt{5}-9\sqrt{2} &= \left(6\sqrt{5}+2\sqrt{5}\right)+\left(9\sqrt{2}-9\sqrt{2}\right)\\ &= (6+2)\sqrt{5}+(9-9)\sqrt{2}\\ &= 8\sqrt{5}+0\sqrt{2}\\ &= 8\sqrt{5}\end{aligned}$$

Use the commutative property to bring the like terms together.

- $\sqrt{11}-4\sqrt{3}-\left(7\sqrt{11}-7\sqrt{3}\right)$

Solution:

$$\begin{aligned}\sqrt{11}-4\sqrt{3}-\left(7\sqrt{11}-7\sqrt{3}\right) &= \sqrt{11}-4\sqrt{3}-7\sqrt{11}+7\sqrt{3}\\ &= \sqrt{11}-7\sqrt{11}-4\sqrt{3}+7\sqrt{3}\\ &= (1-7)\sqrt{11}+(-4+7)\sqrt{3}\\ &= -6\sqrt{11}+3\sqrt{3} \text{ or } 3\sqrt{3}-6\sqrt{11}\end{aligned}$$

The number $\sqrt{3}$ has a coefficient of 1, just like the term z has a coefficient of 1.

The term $0\sqrt{2}$ means $0\times\sqrt{2}$ which is zero. This is why there is no $\sqrt{2}$ term in the sum.

Remember: Change all the signs within parentheses when there is a negative sign in front of the parentheses.

In many expressions, entire radicals must be changed to mixed radicals before the like radicals can be identified and grouped. You can also say that the radicals should be simplified before they are added or subtracted.

Example 4

- Simplify $\sqrt{18}-\sqrt{50}+\sqrt{98}$.

  Solution:

$$\begin{aligned}\sqrt{18}-\sqrt{50}+\sqrt{98} &= \sqrt{9\times 2}-\sqrt{25\times 2}+\sqrt{49\times 2}\\ &\left.\begin{aligned}&= 3\sqrt{2}-5\sqrt{2}+7\sqrt{2}\\ &= (3-5+7)\sqrt{2}\end{aligned}\right\}\text{ distributive property}\\ &= 5\sqrt{2}\end{aligned}$$

- Simplify $\sqrt{12}-\sqrt{98}+\sqrt{32}-\sqrt{75}$.

  Solution:

$$\begin{aligned}\sqrt{12}-\sqrt{98}+\sqrt{32}-\sqrt{75} &= \sqrt{4\times 3}-\sqrt{49\times 2}+\sqrt{16\times 2}-\sqrt{25\times 3}\\ &= 2\sqrt{3}-7\sqrt{2}+4\sqrt{2}-5\sqrt{3}\\ &= 2\sqrt{3}-5\sqrt{3}-7\sqrt{2}+4\sqrt{2}\\ &= (2-5)\sqrt{3}+(-7+4)\sqrt{2}\\ &= -3\sqrt{3}-3\sqrt{2}\end{aligned}$$

Think: Express entire radicals as mixed radicals in simplest form. Try to get as many like radicands as possible.

You are now ready to try some problems on your own.

Do questions 1a, 1c, 2b, 3, 4a, 4c, 5, 6a, and 7. If you need more practice, go back and do the other questions.

1. Simplify each of the following.

   a. $6\sqrt{7}+5\sqrt{7}$

   b. $-8\sqrt{3}+12\sqrt{3}$

   c. $-5\sqrt{22}-8\sqrt{22}$

2. Simplify each of the following.

   a. $7\sqrt{5}+9-2\sqrt{5}-4$

   b. $-6\sqrt{10}+20-\left(4\sqrt{10}+11\right)$

3. a. What would you do first to simplify $\sqrt{48}+\sqrt{75}$?

   b. Rewrite, in simplest form, the radical expression in 3a. Show all your work.

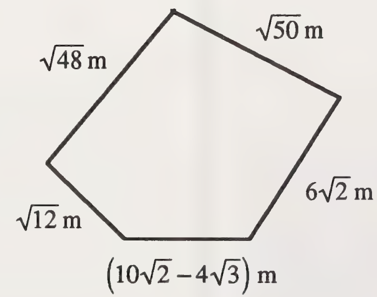
4. Express each of the following in simplest form.

   a. $6\sqrt{2}+\sqrt{32}$

   b. $5\sqrt{2}-\sqrt{98}$

   c. $\sqrt{96}-\sqrt{24}$

5. a. Find the distance around the edges of the following figure. Give the perimeter in simplest radical form.



   b. What is the distance expressed to the nearest hundredth of a metre?

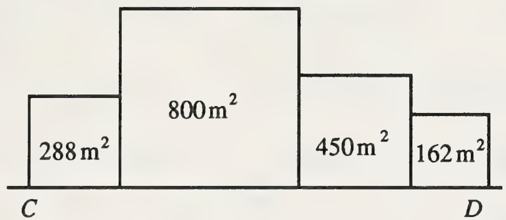
6. Where necessary, change the expressions to mixed radicals and express in simplest form.

   a. $\sqrt{5}+\sqrt{20}-2\sqrt{5}$

   b. $\sqrt{108}-\sqrt{48}+\sqrt{300}$

7. a. In the following diagram, four square lots are illustrated. They are side by side down a street. Find the distance from C to D written in the simplest form possible.

   b. Express your answer to the nearest metre.



For solutions to **Activity 3**, turn to the **Appendix, Topic 1**.

If you require help, do the Extra Help section.
If you want more challenging explorations, do the Extensions section.
} You may decide to do both.

Extra Help

There are two types of radicals.

- Entire radicals

  These expressions are entirely under the radical sign.

  $\sqrt{17}$

  $\sqrt{53}$

- Mixed radicals

  These expressions have a numerical coefficient before the radical sign and a number under the radical sign.

  $4\sqrt{7}$

  $16\sqrt{39}$

Entire radicals can be changed to mixed radicals if a perfect square factor can be removed from the radicand which is the number under the radical sign.

Removing the largest possible square factor from the radicand is called simplifying the radical. If you have trouble identifying the perfect square factor, express the radicand in prime factorization form and use the resulting pairs of common factors to determine the largest possible perfect square value. Note the following examples.

Example 5

Change each entire radical to a mixed radical.

- $\sqrt{50}$

  Solution:

  $$\begin{aligned}\sqrt{50} &= \sqrt{5\times 5\times 2}\\ &= 5\sqrt{2}\end{aligned}$$

- $\sqrt{72}$

  Solution:

  $$\begin{aligned}\sqrt{72} &= \sqrt{2\times 2\times 2\times 3\times 3}\\ &= 2\times 3\sqrt{2}\\ &= 6\sqrt{2}\end{aligned}$$

- $\sqrt{3240}$

  Solution:

$$\begin{aligned}\sqrt{3240} &= \sqrt{(2\times 2)\times 2\times(3\times 3)\times(3\times 3)\times 5}\\ &= 2\times 3\times 3\sqrt{2\times 5}\\ &= 18\sqrt{10}\end{aligned}$$

To change a mixed second-order radical to an entire radical, use the following steps:

- Square the coefficient. (This is the number in front of the radical sign.)
- Multiply the new value by the radicand.
- Place the resulting product under the radical sign.

Example 6

Change $3\sqrt{3}$ and $7\sqrt{3}$ to entire radicals.

Solution:

$$\begin{aligned}3\sqrt{3} &= \sqrt{3^2\times 3}\\ &= \sqrt{9\times 3}\\ &= \sqrt{27}\end{aligned}$$

$$\begin{aligned}7\sqrt{5} &= \sqrt{7^2\times 5}\\ &= \sqrt{49\times 5}\\ &= \sqrt{245}\end{aligned}$$

When adding or subtracting radicals, use the following steps:

- When possible, change the entire radicals to mixed radicals.
- Group like radicals using the commutative property. Like radicals have the same radicands.
- Add or subtract the coefficients of the like radicals.

Example 7

- Simplify $2\sqrt{3}-5\sqrt{3}+7\sqrt{3}$.

  Solution:

$$\begin{aligned}2\sqrt{3}-5\sqrt{3}+7\sqrt{3} &= (2-5+7)\sqrt{3}\\ &= 4\sqrt{3}\end{aligned}$$

Prime factorization for 3240 is found following these steps:

$$\begin{aligned}3240 &= 2\times 1620\\ &= 2\times 2\times 810\\ &= 2\times 2\times 2\times 405\\ &= 2\times 2\times 2\times 3\times 135\\ &= 2\times 2\times 2\times 3\times 3\times 45\\ &= 2\times 2\times 2\times 3\times 3\times 3\times 15\\ &= 2\times 2\times 2\times 3\times 3\times 3\times 3\times 5\end{aligned}$$

A second-order radical is a square root.

- Simplify $\sqrt{50}-\sqrt{72}+\sqrt{8}-\sqrt{98}$.

Solution:

$$\begin{aligned}\sqrt{50}-\sqrt{72}+\sqrt{8}-\sqrt{98} &= \sqrt{25\times 2}-\sqrt{36\times 2}+\sqrt{4\times 2}-\sqrt{49\times 2}\\ &= 5\sqrt{2}-6\sqrt{2}+2\sqrt{2}-7\sqrt{2}\\ &= (5-6+2-7)\sqrt{2}\\ &= -6\sqrt{2}\end{aligned}$$

- Simplify $\sqrt{20}-\sqrt{24}+\sqrt{45}+\sqrt{48}-\sqrt{108}$.

Solution:

$$\begin{aligned}\sqrt{20}-\sqrt{24}+\sqrt{45}+\sqrt{48}-\sqrt{108} &= \sqrt{4\times 5}-\sqrt{4\times 6}+\sqrt{9\times 5}+\sqrt{16\times 3}-\sqrt{36\times 3}\\ &= 2\sqrt{5}-2\sqrt{6}+3\sqrt{5}+4\sqrt{3}-6\sqrt{3}\\ &= 2\sqrt{5}+3\sqrt{5}+4\sqrt{3}-6\sqrt{3}-2\sqrt{6}\\ &= (2+3)\sqrt{5}+(4-6)\sqrt{3}-2\sqrt{6}\\ &= 5\sqrt{5}-2\sqrt{3}-2\sqrt{6}\end{aligned}$$

When a radical cannot be grouped with any others, it must be repeated in the sum or the difference.

- Simplify $\sqrt{7}-3\sqrt{9}+4\sqrt{3}+\sqrt{28}+9-\sqrt{48}$.

Solution:

$$\begin{aligned}\sqrt{7}-3\sqrt{9}+4\sqrt{3}+\sqrt{28}+9-\sqrt{48} &= \sqrt{7}-(3\times 3)+4\sqrt{3}+\sqrt{4\times 7}+9-\sqrt{16\times 3}\\ &= \sqrt{7}-9+4\sqrt{3}+2\sqrt{7}+9-4\sqrt{3}\\ &= \sqrt{7}+2\sqrt{7}+4\sqrt{3}-4\sqrt{3}-9+9 \qquad \text{(Group like terms.)}\\ &= (1+2)\sqrt{7}+(4-4)\sqrt{3}+0\\ &= 3\sqrt{7}+0\sqrt{3}+0\\ &= 3\sqrt{7}\end{aligned}$$

The coefficient in $\sqrt{7}$ is 1.

The questions which follow cover all the concepts found in Topic 1. If you have trouble when working with radicals, do all the questions. This will reinforce the skills which are necessary when working with radicals.

Do all the questions.

1. Change each entire radical to a mixed radical. Use prime factorizations if necessary. Show all your work.

   a. $\sqrt{56}$ b. $\sqrt{63}$

   c. $\sqrt{200}$ d. $\sqrt{180}$

   e. $\sqrt{3564}$ f. $\sqrt{12\,600}$

Remember: You can reach prime factorizations step-by-step. For example,

$$\begin{aligned}3564 &= 2\times 1782\\ &= 2\times 2\times 891\\ &= 2\times 2\times 3\times 297\\ &= 2\times 2\times 3\times 3\times 99\\ &= 2\times 2\times 3\times 3\times 3\times 33\\ &= 2\times 2\times 3\times 3\times 3\times 3\times 11\end{aligned}$$

2. Change each mixed radical to an entire radical. Show all your work.

a. $6\sqrt{5}$ b. $11\sqrt{3}$

c. $5\sqrt{10}$ d. $9\sqrt{2}$

e. $15\sqrt{3}$ f. $2\sqrt{7}+4\sqrt{7}$

3. Simplify each of the following radical expressions.

a. $\sqrt{60}+2\sqrt{15}-4\sqrt{15}$

b. $7-2\sqrt{3}+\sqrt{27}+9-5\sqrt{3}+\sqrt{48}$

c. $-3\sqrt{5}+\sqrt{180}-4\sqrt{5}+\sqrt{125}$

For solutions to **Extra Help**, turn to the **Appendix, Topic 1**.

Extensions

When working with radical expressions, you do not have to restrict yourself to the use of numbers. Many radicals contain variables as well as numbers.

$\sqrt{5a}$ $2a\sqrt{7}$ $4y^2\sqrt{13xy}$

Just as for the simpler radicals studied, there are two types of these radicals.

- Entire radicals

$\sqrt{7b}$ $\sqrt{32x^2}$ $\sqrt{8a^3b^4}$

- Mixed radicals

$3\sqrt{5c}$ $5b^2\sqrt{7a}$ $\frac{1}{2}x^3\sqrt{10}$

Just as for the simpler radicals, entire radicals which contain variables can be changed to mixed radicals, and mixed radicals can be changed to entire radicals.

The square root of a variable term represents a real number only when the variable term represents a nonnegative real number. For example, if you take the principal square root of the variable term x, you arrive at the variable expression $\sqrt{x}$. The radical $\sqrt{x}$ represents a real number only when $x \geq 0$.

Note the following:

When $x = 4$, $\sqrt{x} = \sqrt{4} = 2$ which is a real number.
When $x = -4$, $\sqrt{x} = \sqrt{-4}$ which is **not** a real number.
When $x = 7$, $\sqrt{x} = \sqrt{7}$ which is a real number.
When $x = -7$, $\sqrt{x} = \sqrt{-7}$ which is **not** a real number.

Now consider the principal square root of the variable term x^2. The principal square root can be represented by the symbol $\sqrt{x^2}$. At first glance it appears that $\sqrt{x^2}$ should equal x. However, remember that a principal square root must always be positive. Thus, $\sqrt{x^2}$ equals x if x represents a nonnegative real number, but it does **not** equal x if x represents a negative number. Therefore, you must find some way to represent $\sqrt{x^2}$ so that the result will always be positive or zero. To do this, use the absolute value symbol. If you take the absolute value of x, which is written $|x|$, you know that this value is always nonnegative. Thus, you must define the principal square root of x^2 $\left(\text{written as } \sqrt{x^2}\right)$ to be equal to the absolute value of x, which is written $|x|$.

For any variable x whose domain is R, $\sqrt{x^2} = |x|$.

The previous rule guarantees that the principal square root of a variable term will always be positive or zero, regardless of the value substituted for the variable. For example,

if $x = 5, \sqrt{x^2} = \sqrt{(5)^2} = |5| = 5$

if $x = -5, \sqrt{x^2} = \sqrt{(-5)^2} = |-5| = 5$

Both results are positive.

Any square root whose radicand is a variable term that is a perfect square can be written without the radical sign. An absolute value sign must be placed around the result. Then, any factors that are always positive can be removed from the absolute value sign. Any power with a variable base and an **even** exponent is always positive.

Examples are as follows:

$\sqrt{y^2} = |y|$

$\sqrt{(x+1)^2} = |x+1|$

$\sqrt{9x^2} = \sqrt{(3x)^2} = |3x| = 3|x|$

The factor 3 can be taken outside the absolute value sign since 3 is always positive.

$\sqrt{x^4} = \sqrt{(x^2)^2} = |x^2| = x^2$

The absolute value sign can be dropped here since x^2 is always positive.

$\sqrt{y^6} = \sqrt{(y^3)^2} = |y^3|$

The absolute value sign must be retained since y^3 can be positive or negative.

Example 8

- Change the entire radical $\sqrt{32a^3b^4}$ to a mixed radical.

Solution:

$$\begin{aligned}\sqrt{32a^3b^4} &= \sqrt{2\times2\times2\times2\times2\times a\times a\times a\times b\times b\times b\times b}\\ &= \sqrt{2^2\times2^2\times2\times a^2\times a\times b^2\times b^2}\\ &= 2\times2\times|a|\times|b|\times|b|\times\sqrt{2\times a}\\ &= 4\left|ab^2\right|\sqrt{2\times a}\\ &= 4|a|b^2\sqrt{2a}\end{aligned}$$

An alternate method to do this example is as follows:

$$\begin{aligned}\sqrt{32a^3b^4} &= \sqrt{16a^2b^4(2a)}\\ &= 4\left|ab^2\right|\sqrt{2a}\\ &= 4|a|b^2\sqrt{2a}\end{aligned}$$

- Change the mixed radical $6x^3y\sqrt{3y}$ to an entire radical.

Solution:

$$\begin{aligned}6x^3y\sqrt{3y} &= \sqrt{(6x^3y)^2(3y)}\\ &= \sqrt{36x^6y^2\times3y}\\ &= \sqrt{108x^6y^3}\end{aligned}$$

Radical expressions that have variables can also be added and subtracted. As mentioned earlier, the radicands must be like or common if simplification is to be done using addition and subtraction.

Example 9

- Simplify $3a\sqrt{2b}+7a\sqrt{2b}-5a\sqrt{2b}$.

  Solution:

$$\begin{aligned} 3a\sqrt{2b}+7a\sqrt{2b}-5a\sqrt{2b} &= (3a+7a-5a)\sqrt{2b} \\ &= 5a\sqrt{2b} \end{aligned}$$

- Simplify $\sqrt{72x^2yz}-\sqrt{8x^2yz}+\sqrt{32x^2yz}-\sqrt{8x^2yz}$.

  Solution:

$$\begin{aligned} \sqrt{72x^2yz}-\sqrt{8x^2yz}+\sqrt{32x^2yz}-\sqrt{8x^2yz} &= \sqrt{36x^2(2yz)}-\sqrt{4x^2(2yz)}+\sqrt{16x^2(2yz)}-\sqrt{4x^2(2yz)} \\ &= 6|x|\sqrt{2yz}-2|x|\sqrt{2yz}+4|x|\sqrt{2yz}-2|x|\sqrt{2yz} \\ &= (6|x|-2|x|+4|x|-2|x|)\sqrt{2yz} \\ &= 6|x|\sqrt{2yz} \end{aligned}$$

Note: $\sqrt{x^2}=|x|$

To this point you have used whole numbers or integers as the numerical coefficients in the multiplier and the radicand of a radical expression. This need not be the case at all times. Often these coefficients may be fractional or decimal values.

Examples of entire radicals are as follows:

$$\sqrt{\frac{1}{4}a^2b^3} \qquad \sqrt{0.04x^3y^3}$$

Examples of mixed radicals are as follows:

$$\frac{1}{3}xy\sqrt{3z} \qquad 0.3a\sqrt{bcd}$$

To simplify the radical expressions mentioned, keep in mind that only like or common radicals can be added or subtracted.

Example 10

- Simplify $\frac{2}{5}a\sqrt{7c}+\frac{1}{2}a\sqrt{7c}-\frac{3}{4}a\sqrt{7c}$.

Solution:

$$\begin{aligned}\frac{2}{5}a\sqrt{7c}+\frac{1}{2}a\sqrt{7c}-\frac{3}{4}a\sqrt{7c} &= \left(\frac{2}{5}a+\frac{1}{2}a-\frac{3}{4}a\right)\sqrt{7c}\\ &= \left(\frac{8a+10a-15a}{20}\right)\sqrt{7c}\\ &= \left(\frac{18a-15a}{20}\right)\sqrt{7c}\\ &= \frac{3a}{20}\sqrt{7c}\end{aligned}$$

- Simplify $\sqrt{6.25x^2yz}+\sqrt{2.25x^2yz}-\sqrt{20.25x^2yz}+\sqrt{0.25x^2yz}$.

  Solution:

$$\begin{aligned}\sqrt{6.25x^2yz}+\sqrt{2.25x^2yz}-\sqrt{20.25x^2yz}+\sqrt{0.25x^2yz} &= 2.5|x|\sqrt{yz}+1.5|x|\sqrt{yz}-4.5|x|\sqrt{yz}+0.5|x|\sqrt{yz}\\ &= (2.5|x|+1.5|x|-4.5|x|+0.5|x|)\sqrt{yz}\\ &= 0|x|\sqrt{yz}\\ &= 0\end{aligned}$$

Besides having square-root radicals, you will find that other root radicals can be used.

$\sqrt[3]{8a^6c}$ $\quad$ $\sqrt[4]{64b^7}$

To change these radicals to mixed radicals, proceed as in the next example.

Example 11

- Change $\sqrt[3]{8a^6c}$ to a mixed radical.

  Solution:

$$\begin{aligned}\sqrt[3]{8a^6c} &= \sqrt[3]{2\times2\times2\times a\times a\times a\times a\times a\times a\times c}\\ &= (2a^2)\sqrt[3]{c}\end{aligned}$$

Note: For cube roots, arrange all like factors into groups of three.

Use parentheses to separate the exponent from the cube root.

- Change $\sqrt[4]{64b^7}$ to a mixed radical.

  Solution:

$$\sqrt[4]{64b^7} = \sqrt[4]{2\times2\times2\times2\times2\times2\times b\times b\times b\times b\times b\times b\times b}$$
$$= (2b)\sqrt[4]{4b^3}$$

To change mixed radicals to entire radicals, you do the opposite.

Example 12

- Change $(5x^2)\sqrt[3]{y}$ to an entire radical.

  Solution:

$$(5x^2)\sqrt[3]{y} = \sqrt[3]{5\times5\times5\times x^2\times x^2\times x^2\times y}$$
$$= \sqrt[3]{125x^6y}$$

- Change $(2a)\sqrt[5]{bc}$ to an entire radical.

  Solution:

$$(2a)\sqrt[5]{bc} = \sqrt[5]{2\times2\times2\times2\times2\times a\times a\times a\times a\times a\times b\times c}$$
$$= \sqrt[5]{32a^5bc}$$

Note: For fourth roots, arrange all like factors into groups of four.

To add or subtract these radicals, two conditions for the radicals must be present if they are to be classified as being like or common.

- The radicand must be exactly the same in each term.
- The index must be the same in each term.

Recall:

$$\text{index} \rightarrow \sqrt[3]{\underbrace{27a^7b}_{\text{radicand}}}$$

It is time to do some of these more difficult problems on your own. Doing these problems should help you learn the concepts mentioned in this section.

Do parts a and b of each question. If you require more practice, go back and do part c of each question.

1. Change each of the following to mixed radicals.

   a. $\sqrt{36a^2b^3c^4}$

   b. $\sqrt{\frac{9}{16}x^5y^2}$

   c. $\sqrt{0.16c^6d^3}$

2. Change each of the following to entire radicals.

   a. $5xyz\sqrt{2x}$

   b. $\frac{2}{5}a^2b\sqrt{10c}$

   c. $1.3c^3\sqrt{6d}$

3. Simplify each of the following.

   a. $4cd\sqrt{2f}+3cd\sqrt{2f}-5cd\sqrt{2f}-9cd\sqrt{2f}$

   b. $-\sqrt{0.18x^3y}+\sqrt{5.12x^3y}-\sqrt{18x^3y}+\sqrt{32x^3y}$

For solutions to **Extensions**, turn to the **Appendix**, **Topic 1**.

Topic 2 Multiplying and Dividing Radicals

Introduction

In the last topic you learned that radical expressions can be added or subtracted. Several comparisons were made to show how these operations are applied to radical expressions and polynomials. The similarities do not stop with addition and subtraction.

You will see that the multiplication and division of radicals are closely related to the multiplication and division of algebraic or polynomial expressions. Various formulas used in industry are based on the multiplication and division of radical expressions.

What Lies Ahead

Throughout the topic you will learn to

1. multiply radicals
2. multiply radicals which are conjugates of one another
3. divide radicals

Now that you know what to expect, turn the page to begin your study of multiplying and dividing radicals.

Exploring Topic 2

Activity 1

Multiply radicals.

As mentioned previously, different methods often exist for performing the same calculation. Look at the following example.

Example 1

Multiply $\sqrt{4} \times \sqrt{9}$.

Solution:

$$\sqrt{4} \times \sqrt{9} = 2 \times 3$$
$$= 6$$

Here the square roots are found first; then the resulting values are multiplied.

OR

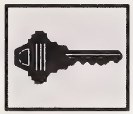
$$\sqrt{4} \times \sqrt{9} = \sqrt{4 \times 9}$$
$$= \sqrt{36}$$
$$= 6$$

Here the radicands are multiplied first; then the square root of the resulting product is taken.

To change the mixed radical $5\sqrt{2}$ to an entire radical, you square the 5 to get $\sqrt{25}$, and then multiply by $\sqrt{2}$ to get $\sqrt{50}$ which is an entire radical.

$$5\sqrt{2} = \sqrt{5 \times 5} \times \sqrt{2}$$
$$= \sqrt{25} \times \sqrt{2}$$
$$= \sqrt{25 \times 2}$$
$$= \sqrt{50}$$

The main property involved in multiplying radicals is as follows:



When $a > 0$ and $b > 0$, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$. This means that when you multiply two or more entire radicals, simply multiply the radicands.

Example 2

Multiply $\sqrt{2} \times \sqrt{3}$.

Solution:

$$\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3}$$
$$= \sqrt{6}$$

In many instances further simplification of the product is needed.

You should use the method of calculation which you find easiest.

Example 3

Multiply $\sqrt{3} \times \sqrt{18}$ and simplify.

Solution:

$$\begin{aligned} \sqrt{3} \times \sqrt{18} &= \sqrt{3 \times 18} \\ &= \sqrt{54} \\ &= \sqrt{9 \times 6} \\ &= \sqrt{9} \times \sqrt{6} \\ &= 3 \times \sqrt{6} \\ &= 3\sqrt{6} \end{aligned}$$

OR

$$\begin{aligned} \sqrt{3} \times \sqrt{18} &= \sqrt{3} \times \sqrt{9 \times 2} \\ &= \sqrt{3} \times \sqrt{9} \times \sqrt{2} \\ &= 3 \times \sqrt{3} \times \sqrt{2} \\ &= 3\sqrt{3 \times 2} \\ &= 3\sqrt{6} \end{aligned}$$

See the margin box for a different approach to solving this example.

To find the product of two or more mixed radicals, multiply the coefficients or multipliers first. Then multiply the radicals by multiplying the radicands.

Example 4

Multiply $2\sqrt{5} \times 4\sqrt{3}$ and simplify.

Solution:

$$\begin{aligned} 2\sqrt{5} \times 4\sqrt{3} &= 2 \times 4 \times \sqrt{5} \times \sqrt{3} \\ &= 8\sqrt{5 \times 3} \\ &= 8\sqrt{15} \end{aligned}$$

In the example which follows, notice that skills used in multiplying polynomials are used in multiplying more complicated radical expressions. Take note of the similarities that exist.

Example 5

- Multiply the polynomials $2a$ and $3b$.

Solution:

$$\begin{aligned} (2a)(3b) &= 2 \times 3 \times a \times b \\ &= 6ab \end{aligned}$$

(You are multiplying two monomials here.)

The product $8\sqrt{15}$ is in simplest form. This is not the case in all instances.

Note a different approach to Example 3:
This calculation can also be done as follows:

$$\begin{aligned} \sqrt{3} \times \sqrt{18} &= \sqrt{3} \times \sqrt{3 \times 6} \\ &= \sqrt{3} \times \sqrt{3} \times \sqrt{6} \\ &= 3\sqrt{6} \end{aligned}$$

Notice that the product is in simplest form.

Recall:

$3\sqrt{21}$

multiplier radicand

- Multiply the radicals $2\sqrt{3}$ and $3\sqrt{5}$.

Solution:

$$\begin{aligned}\left(2\sqrt{3}\right)\left(3\sqrt{5}\right) &= 2\times 3\times\sqrt{3}\times\sqrt{5}\\ &= 6\sqrt{3\times 5}\\ &= 6\sqrt{15}\end{aligned}$$

- Multiply the polynomials $4b$ and $(b-3)$.

Solution:

Use the distributive property $a(b-c)=ab-ac$.

$$\begin{aligned}4b(b-3) &= 4b(b)-4b(3)\\ &= 4b^2-12b\end{aligned}$$

- Multiply the radicals $6\sqrt{3}$ and $\left(\sqrt{5}-2\right)$.

Solution:

In radical form, 2 can be written as $2\sqrt{1}$.

$$\begin{aligned}6\sqrt{3}\left(\sqrt{5}-2\right) &= \left(6\sqrt{3}\right)\left(\sqrt{5}\right)-\left(6\sqrt{3}\right)(2)\\ &= \left(6\sqrt{3}\right)\left(\sqrt{5}\right)-\left(6\sqrt{3}\right)\left(2\sqrt{1}\right)\\ &= 6\sqrt{15}-12\sqrt{3}\end{aligned}$$

The expression cannot be simplified any further.

- Multiply the polynomials $(m+4)$ and $(m-2)$.

Solution:

$$\begin{aligned}&(m+4)(m-2)\\ &=(m)(m)-(m)(2)+(4)(m)+(4)(-2)\\ &=m^2-2m+4m-8\\ &=m^2+2m-8\end{aligned}$$

- Multiply the radicals $\left(\sqrt{5}+4\right)$ and $\left(\sqrt{5}-2\right)$.

Solution:

$$\begin{aligned}&\left(\sqrt{5}+4\right)\left(\sqrt{5}-2\right)\\ &=\left(\sqrt{5}\right)\left(\sqrt{5}\right)+\left(\sqrt{5}\right)(-2)+(4)\left(\sqrt{5}\right)+(4)(-2)\\ &=5-2\sqrt{5}+4\sqrt{5}-8\\ &=-3+2\sqrt{5}\ \text{ or }\ 2\sqrt{5}-3\end{aligned}$$

Recall: Use FOIL to help you multiply binomial expressions.

$$(m+4)(m-2)$$

First $(m)(m)=m^2$

Outer $(m)(-2)=-2m$

Inner $(4)(m)=4m$

Last $(4)(-2)=-8$

Therefore,

$(m+4)(m-2)=m^2+2m-8.$

You will notice in the last two examples that the product of the algebraic binomials is a trinomial, but the product of two binomials involving radicals is a binomial. Although this is not always the case, it does happen quite often since any radical multiplied by itself results in a whole number.

For example,

$$2\sqrt{5}\times\sqrt{5}=2\times5 \qquad 6\sqrt{7}\times2\sqrt{7}=6\times2\times7$$
$$=10 \qquad\qquad =84$$

Now it is time to do some questions on your own.

Do the odd-numbered questions. If you require more practice, do all of the questions.

Express each product in simplest form:

1. $\sqrt{3}\times\sqrt{6}$
2. $\sqrt{8}\times\sqrt{6}$
3. $4\sqrt{5}\times\sqrt{3}$
4. $7\sqrt{2}\times\sqrt{6}$
5. $2\sqrt{3}\times4\sqrt{8}$
6. $5\sqrt{5}\times3\sqrt{8}$
7. $\sqrt{3}\left(\sqrt{3}+\sqrt{8}\right)$
8. $\sqrt{11}\left(\sqrt{11}+\sqrt{8}\right)$
9. $10\left(8\sqrt{2}-\sqrt{6}\right)$
10. $3\left(10\sqrt{5}-\sqrt{14}\right)$
11. $3\sqrt{3}\left(5\sqrt{6}+2\sqrt{5}\right)$
12. $2\sqrt{5}\left(4\sqrt{7}+3\sqrt{6}\right)$
13. $\left(3\sqrt{5}-4\right)\left(3+2\sqrt{3}\right)$
14. $\left(5\sqrt{2}-6\right)\left(5+4\sqrt{2}\right)$
15. $\left(4\sqrt{3}+\sqrt{6}\right)^2$
16. $\left(3\sqrt{2}+\sqrt{5}\right)^2$

For solutions to **Activity 1**, turn to the **Appendix, Topic 2**.

Activity 2

Multiply radicals which are conjugates of one another.

From your work with polynomials, you should remember the difference of squares which results from the multiplication of two binomials of the form $(a-b)$ and $(a+b)$.

$$\begin{aligned}(a+b)(a-b)&=a^2-ab+ab-b^2\\&=a^2-b^2\end{aligned}$$

This same pattern results when radical binomials of a corresponding form are multiplied.

The sum of ab and $-ab$ is zero. Such values are called additive inverses.

Example 6

Multiply $\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)$.

Solution:

$$\begin{aligned}\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)&=\sqrt{5}\times\sqrt{5}+\sqrt{5}\times-\sqrt{2}+\sqrt{2}\times\sqrt{5}+\sqrt{2}\times-\sqrt{2}\\&=5-\sqrt{10}+\sqrt{10}-2\\&=5-2\\&=3\end{aligned}$$

Recall: $\sqrt{5}\times\sqrt{5}=\left(\sqrt{5}\right)^2$
$=5$

This pattern is important to remember since the resulting simplified product is not a radical expression.

Binomials of the form $(a + b)$ and $(a - b)$ are called **conjugates**. For example, $\left(2\sqrt{5} - 3\right)$ and $\left(2\sqrt{5} + 3\right)$ are conjugates of each other. Note that the product of conjugates involving radicals results in a real number.

Do the odd-numbered questions. If you want more practice, do the remaining questions.

Multiply the given radical expression by its conjugate.

1. $\left(\sqrt{2} - \sqrt{3}\right)$
2. $\left(\sqrt{6} + \sqrt{5}\right)$
3. $\left(3\sqrt{3} - \sqrt{5}\right)$
4. $\left(4\sqrt{6} + \sqrt{2}\right)$
5. $\left(\sqrt{5} - 2\sqrt{7}\right)$
6. $\left(\sqrt{6} + 4\sqrt{2}\right)$
7. $\left(6\sqrt{3} - 2\sqrt{10}\right)$
8. $\left(5\sqrt{13} + 2\sqrt{11}\right)$

For solutions to **Activity 2**, turn to the **Appendix, Topic 2**.

Activity 3

Divide radicals.

When you did multiplication of radical expressions, you saw that different approaches can be used. This situation also applies to the division of radicals.

Study the following:

$$\frac{\sqrt{100}}{\sqrt{25}} = \frac{10}{5} = 2 \qquad \text{or} \qquad \sqrt{\frac{100}{25}} = \sqrt{4} = 2$$

In general, the following relationship is true:

If $x > 0$ and $y > 0$, then

$$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}.$$

This division property of radicals can be used to simplify quotients involving radicals.

Use the shortcut to save work.

$$(x+y)(x-y) = x^2 - y^2$$

Recall: If $x < 0$ or $y < 0$, then the whole radical expression would be undefined since the square root of a negative number is undefined.

Example 7

- Simplify $\frac{\sqrt{27}}{\sqrt{3}}$.

Solution:

$$\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{27}{3}} \qquad \text{or} \qquad \frac{\sqrt{27}}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{3}}$$
$$= \sqrt{9} \qquad\qquad = 3$$
$$= 3$$

- Simplify $\frac{63\sqrt{98}}{7\sqrt{2}}$.

Solution:

$$\begin{aligned}\frac{63\sqrt{98}}{7\sqrt{2}} &= \frac{63}{7}\sqrt{\frac{98}{2}}\\ &= 9\sqrt{49}\\ &= 9\times 7\\ &= 63\end{aligned}$$

- Simplify $\frac{30\sqrt{168}}{10\sqrt{14}}$.

Solution:

$$\begin{aligned}\frac{30\sqrt{168}}{10\sqrt{14}} &= \frac{30}{10}\sqrt{\frac{168}{14}}\\ &= 3\sqrt{12}\\ &= 3\sqrt{4\times 3}\\ &= 3\times 2\sqrt{3}\\ &= 6\sqrt{3}\end{aligned}$$

Remember that $\sqrt{12}$ can be simplified since it has a factor which is a perfect square. You should always simplify as much as possible.

The division property $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ does not always help to simplify an expression or lead to a quotient in simplest form.

Take an expression such as $\frac{\sqrt{3}}{\sqrt{5}}$. Direct division is not possible since 5 does not divide evenly into 3. The only way to simplify a quotient such as this is to eliminate the radical in the denominator. This process is called **rationalizing the denominator**.

When working with radical quotients, the instructions **simplify** and **rationalize the denominator** mean the same thing.

In $\frac{\sqrt{3}}{\sqrt{5}}$, the radical in the denominator can be eliminated by multiplying the expression by $\frac{\sqrt{5}}{\sqrt{5}}$.

$$\begin{aligned}\frac{\sqrt{3}}{\sqrt{5}} &= \frac{\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{\sqrt{15}}{\sqrt{25}} \\ &= \frac{\sqrt{15}}{5}\end{aligned}$$

For an even better understanding, study the more difficult examples that follow.

Example 8

Simplify $\frac{\sqrt{7}}{3\sqrt{2}}$.

Solution:

$$\begin{aligned}\frac{\sqrt{7}}{3\sqrt{2}} &= \frac{\sqrt{7}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{14}}{3 \times 2} \\ &= \frac{\sqrt{14}}{6}\end{aligned}$$

Example 9

Rationalize the denominator in $\frac{2\sqrt{5} - \sqrt{2}}{\sqrt{3}}$.

Solution:

$$\begin{aligned}\frac{2\sqrt{5} - \sqrt{2}}{\sqrt{3}} &= \frac{2\sqrt{5} - \sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{\left(2\sqrt{5}\right)\left(\sqrt{3}\right) - \left(\sqrt{2}\right)\left(\sqrt{3}\right)}{\left(\sqrt{3}\right)\left(\sqrt{3}\right)} \\ &= \frac{2\sqrt{15} - \sqrt{6}}{3}\end{aligned}$$

You are now ready to try some problems on your own. If you want more help on this topic, do the **Extra Help** section. If you want more challenging explorations, do the **Extensions** section.

Recall: Simplifying an expression by multiplying by 1 or an equivalent form of 1 such as $\frac{\sqrt{5}}{\sqrt{5}}$ may change the form of the expression, but it does not change its value.

$$\frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$$

This is the unsimplified quotient.

This is considered to be a simplified form since there is no radical in the denominator.

It is not necessary to multiply by $\frac{3\sqrt{2}}{3\sqrt{2}}$ since the radical disappears when you multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

Do questions 1a, 1c, 2a, 2c, and 3. If you think you need more practice, do the remaining questions.

1. Simplify.

   a. $\dfrac{15\sqrt{45}}{3\sqrt{5}}$ b. $\dfrac{30\sqrt{50}}{6\sqrt{2}}$

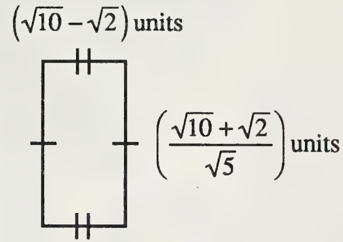
   c. $\dfrac{4\sqrt{6}}{\sqrt{3}}$ d. $\dfrac{14\sqrt{22}}{\sqrt{8}}$

2. Rationalize the denominator in each of the following.

   a. $\dfrac{2\sqrt{6}+10}{\sqrt{3}}$ b. $\dfrac{6\sqrt{3}-7}{\sqrt{5}}$

   c. $\dfrac{\sqrt{5}-\sqrt{2}}{3\sqrt{2}}$ d. $\dfrac{\sqrt{10}+\sqrt{6}}{5\sqrt{3}}$

3. Find an expression for the area of the figure shown. Express your solution in simplest form by rationalizing the denominator.



For solutions to **Activity 3**, turn to the **Appendix, Topic 2**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

You may decide to do both.

Extra Help

When multiplying and dividing radical expressions, all that you must remember is the following order:

- Multiply or divide the multipliers.
- Multiply or divide the radicals.
- Simplify the product or quotient to complete the solution.

Example 10

- Multiply $4\sqrt{3}$ and $2\sqrt{6}$.

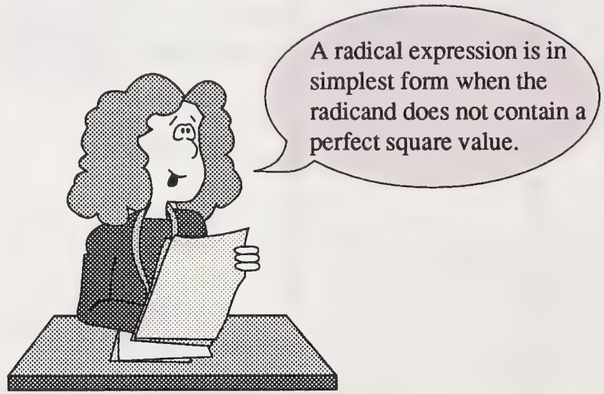
Solution:

$$\begin{aligned} 4\sqrt{3} \times 2\sqrt{6} &= 4 \times 2 \times \sqrt{3} \times \sqrt{6} \\ &= 8 \times \sqrt{3 \times 6} \\ &= 8\sqrt{18} \\ &= 8\sqrt{9 \times 2} \\ &= 24\sqrt{2} \end{aligned}$$

- Find the quotient of $\frac{20\sqrt{30}}{10\sqrt{3}}$.

Solution:

$$\begin{aligned} \frac{20\sqrt{30}}{10\sqrt{3}} &= \frac{20}{10} \times \frac{\sqrt{30}}{\sqrt{3}} \\ &= 2 \times \sqrt{\frac{30}{3}} \\ &= 2\sqrt{10} \end{aligned}$$



Example 11

- Find the simplest form of $3\sqrt{50}$.

Solution:

The radical $3\sqrt{50}$ is not in simplest form because 50 can be expressed as 25×2 and 25 is a perfect square.

$$\begin{aligned} 3\sqrt{50} &= 3\sqrt{25 \times 2} \\ &= 3\sqrt{5^2 \times 2} \\ &= 3 \times 5\sqrt{2} \\ &= 15\sqrt{2} \end{aligned}$$

- Find the simplest form of $6\sqrt{14}$.

Solution:

The radical $6\sqrt{14}$ is in simplest form since 14 does not contain a perfect square factor.

Radical expressions which have radicals in the denominator are not in simplest form. If the denominator is a single term or monomial expression, multiply the radical by itself to simplify. Remember to multiply the numerator by the same value. This form of simplification is called rationalizing the denominator.

Example 12

- Simplify $\frac{2\sqrt{3}}{\sqrt{6}}$.

Solution:

$$\begin{aligned} \frac{2\sqrt{3}}{\sqrt{6}} &= \frac{2\sqrt{3}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{2\sqrt{18}}{6} \\ &= \frac{2\sqrt{9 \times 2}}{6} \\ &= \frac{2 \times 3\sqrt{2}}{6} \\ &= \frac{6\sqrt{2}}{6} \\ &= \sqrt{2} \end{aligned}$$

Recall: Perfect squares are 1, 4, 9, 16, 25, 36, 49, ...

Recall:

$$\frac{\sqrt{5}}{\sqrt{5}} = 1 \qquad \frac{\sqrt{6}}{\sqrt{6}} = 1$$

This means that you are really multiplying by 1, which does not change the original value.

In Example 12,

$$\begin{aligned} \frac{6\sqrt{2}}{6} &= \frac{6}{6} \times \frac{\sqrt{2}}{1} \\ &= 1 \times \sqrt{2} \\ &= \sqrt{2} \end{aligned}$$

- Simplify $\dfrac{4\sqrt{6}}{3\sqrt{2}}$.

Solution:

$$\begin{aligned}\frac{4\sqrt{6}}{3\sqrt{2}} &= \frac{4\sqrt{6}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{4\sqrt{12}}{3\times 2} \\ &= \frac{4\sqrt{4\times 3}}{6} \\ &= \frac{4\times 2\sqrt{3}}{6} \\ &= \frac{8\sqrt{3}}{6} \\ &= \frac{4\sqrt{3}}{3}\end{aligned}$$

Notice that in all of the simplified forms the denominator either disappears or does not have a radical in it.

For a chance to apply what you have learned, do the questions that follow.

Do questions a, c, and e in each section. If you want more practice, go back and do the remaining questions.

1. Multiply and express the products in simplest form.

   a. $\sqrt{6} \times \sqrt{2}$

   b. $\sqrt{5} \times \sqrt{6} \times \sqrt{2}$

   c. $4\sqrt{3} \times 2\sqrt{6}$

   d. $2\sqrt{2} \times 3\sqrt{5} \times 6\sqrt{2}$

   e. $5\sqrt{10} \times 4\sqrt{5} \times 3\sqrt{2}$

2. Divide and express the quotient in simplest form.

   a. $\dfrac{\sqrt{10}}{\sqrt{2}}$

   b. $\dfrac{\sqrt{24}}{\sqrt{3}}$

   c. $\dfrac{3\sqrt{50}}{\sqrt{2}}$

   d. $\dfrac{16\sqrt{72}}{4\sqrt{8}}$

   e. $\dfrac{6\sqrt{10}+2\sqrt{6}}{2\sqrt{2}}$

When $\frac{8\sqrt{3}}{6}$ becomes $\frac{4\sqrt{3}}{3}$, the fraction $\frac{8}{6}$ is reduced.

To rationalize the denominator $2\sqrt{2}$, multiply by only $\sqrt{2}$ in the denominator and numerator since $2\sqrt{2} \times \sqrt{2}$ becomes $2 \times 2 = 4$.

3. Simplify each of the following expressions by rationalizing the denominator.

a. $\frac{3}{\sqrt{2}}$

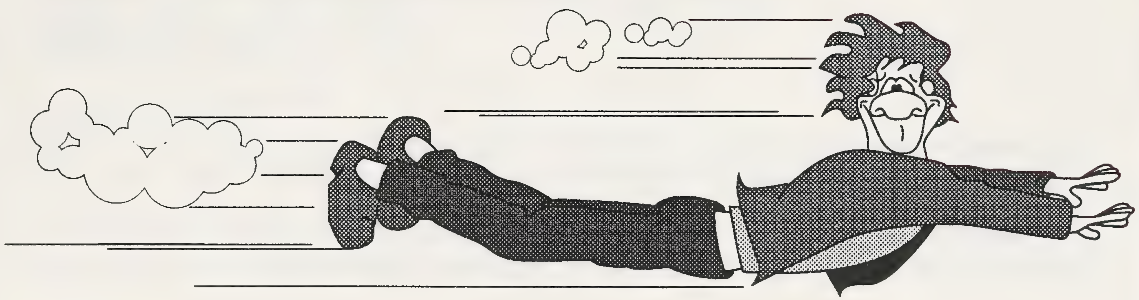
b. $\frac{5}{\sqrt{5}}$

c. $\frac{\sqrt{3}}{\sqrt{5}}$

d. $\frac{2\sqrt{5}}{\sqrt{3}}$

e. $\frac{2\sqrt{6}}{\sqrt{3}}$

For solutions to **Extra Help**, turn to the **Appendix, Topic 2**.



Extensions

Earlier you saw that radicals containing variables can be changed from entire radicals to mixed radicals and from mixed radicals to entire radicals. You also learned that all radicals can be added and subtracted.

Here you will see that radicals with variables can be multiplied and divided. They also have conjugates which are used to rationalize denominators. In these cases the denominator may still have variables, but it will not have a radical.

To multiply, follow these steps.

- Multiply the portion outside the radical sign (often called the multiplier).
- Multiply the radicands.
- Simplify by removing all perfect square factors from the radicand.

Example 13

- Multiply and simplify $3a\sqrt{2c} \times 4a\sqrt{3c} \times 2a\sqrt{2c}$.

  Solution:

$$\begin{aligned} 3a\sqrt{2c} \times 4a\sqrt{3c} \times 2a\sqrt{2c} &= 3a \times 4a \times 2a \times \sqrt{2c} \times \sqrt{3c} \times \sqrt{2c} \\ &= 24a^3\sqrt{12c^3} \\ &= 24a^3\sqrt{4 \times 3 \times c^2 \times c} \\ &= 48a^3c\sqrt{3c} \end{aligned}$$

- Multiply and simplify $2a^2b\sqrt{ab} \times 4ab\sqrt{2abc} \times a^3c\sqrt{6ab}$.

  Solution:

$$\begin{aligned} 2a^2b\sqrt{ab} \times 4ab\sqrt{2abc} \times a^3c\sqrt{6ab} &= 2a^2b \times 4ab \times a^3c \times \sqrt{ab} \times \sqrt{2abc} \times \sqrt{6ab} \\ &= 8a^6b^2c\sqrt{12a^3b^3c} \\ &= 8a^6b^2c\sqrt{4 \times 3 \times a^2 \times b^2 \times a \times b \times c} \\ &= 16a^7b^3c\sqrt{3abc} \end{aligned}$$

The same steps apply to the division operation. Divide the multipliers, then divide the radicands.

Example 14

- Divide and simplify $\dfrac{16a^3b^2\sqrt{10ab}}{2ab\sqrt{5b}}$.

  Solution:

$$\begin{aligned} \frac{16a^3b^2\sqrt{10ab}}{2ab\sqrt{5b}} &= \frac{16a^3b^2}{2ab} \times \frac{\sqrt{10ab}}{\sqrt{5b}} \\ &= 8a^2b\sqrt{2a} \end{aligned}$$

- Divide and simplify $\dfrac{4x^2y^2\sqrt{14xy}}{2y\sqrt{7y}}$.

Solution:

$$\frac{4x^2y^2\sqrt{14xy}}{2y\sqrt{7y}}=\frac{4x^2y^2}{2y}\times\frac{\sqrt{14xy}}{\sqrt{7y}}$$
$$=2x^2y\sqrt{2x}$$

You learned previously that a radical expression is not in simplest form if the denominator has a radical. To simplify in this case, you must rationalize the denominator.

Example 15

- Simplify $\dfrac{3a^2\sqrt{b}}{\sqrt{3c}}$.

Solution:

$$\frac{3a^2\sqrt{b}}{\sqrt{3c}}=\frac{3a^2\sqrt{b}}{\sqrt{3c}}\times\frac{\sqrt{3c}}{\sqrt{3c}}$$
$$=\frac{3a^2\sqrt{3bc}}{3c}$$
$$=\frac{a^2\sqrt{3bc}}{c},\ c\neq 0$$

- Simplify $\dfrac{4a}{\sqrt{2a}+\sqrt{3b}}$.

Solution:

In this case you can rationalize the denominator by multiplying the numerator and the denominator by the conjugate of the denominator.

$$\frac{4a}{\sqrt{2a}+\sqrt{3b}}=\frac{4a}{\sqrt{2a}+\sqrt{3b}}\times\frac{\sqrt{2a}-\sqrt{3b}}{\sqrt{2a}-\sqrt{3b}}$$
$$=\frac{4a\sqrt{2a}-4a\sqrt{3b}}{2a-3b},\ a\neq\frac{3b}{2}$$

- Simplify $\dfrac{2\sqrt{x}-5}{3\sqrt{x}+10}$.

Solution:

$$\frac{2\sqrt{x}-5}{3\sqrt{x}+10}=\frac{2\sqrt{x}-5}{3\sqrt{x}+10}\times\frac{3\sqrt{x}-10}{3\sqrt{x}-10}$$
$$=\frac{6\sqrt{x}^2-20\sqrt{x}-15\sqrt{x}+50}{9x-100}$$
$$=\frac{6x-35\sqrt{x}+50}{9x-100},\ x\neq\frac{100}{9}$$

The conjugate of the denominator is $\sqrt{2a}-\sqrt{3b}$.

These expressions are defined for all replacements of the variables except for any replacements for which the value of the denominator becomes zero. If the denominator is zero, the expression is undefined. Therefore, you must state restrictions on the variables. These restrictions are called nonpermissible values.

Now try some of these more complex problems.

Do problems b, d, and f in the following three questions. If you want more practice, do the remaining parts of the questions.

1. For each of the following, find the product and simplify the final answer.

   a. $2\sqrt{a}\left(2\sqrt{a}-5\sqrt{ay}\right)$

   b. $\left(5\sqrt{x}-\sqrt{3y}\right)\left(\sqrt{4x}+\sqrt{5y}\right)$

   c. $\left(4\sqrt{x}-\sqrt{y}+\sqrt{z}\right)\left(4\sqrt{x}+\sqrt{y}-\sqrt{z}\right)$

   d. $\left(\sqrt{x}-\sqrt{y}\right)\left(\sqrt{x}+\sqrt{y}\right)(x+y)\left(x^2+y^2\right)$

   e. $\left(5\sqrt{x}-2\sqrt{y}\right)\left(3\sqrt{x}+5\sqrt{y}\right)$

   f. $\left(3\sqrt{5x}+2\sqrt{3y}\right)\left(3\sqrt{5x}-2\sqrt{3y}\right)$

2. Divide each of the following. Express the quotients in simplest form.

   a. $\dfrac{16x^2\sqrt{7}}{4x}$

   b. $\dfrac{50a^3\sqrt{21}}{5a\sqrt{7}}$

   c. $\dfrac{72b^4\sqrt{27b^2}}{9b^5}$

   d. $\dfrac{100x\sqrt{50y^3}}{\sqrt{50y^2}}$

   e. $\dfrac{2abc\sqrt{72}}{16ac}$

   f. $\dfrac{\sqrt{1250x^3y^4}}{10xy^2}$

3. Simplify each of the following by rationalizing the denominator.

   a. $\dfrac{3\sqrt{a}}{\sqrt{3}}$

   b. $\dfrac{5}{\sqrt{x}+1}$

   c. $\dfrac{\sqrt{5}}{\sqrt{3}+\sqrt{x}}$

   d. $\dfrac{24}{3\sqrt{a}-5\sqrt{2}}$

   e. $\dfrac{4+3\sqrt{a}}{-6-2\sqrt{a}}$

   f. $\dfrac{3\sqrt{y}-2\sqrt{x}}{4\sqrt{y}-3\sqrt{x}}$

For solutions to **Extensions**, turn to the **Appendix**, **Topic 2**.

Topic 3 Solving and Applying Radical Equations

Introduction

The shape of a roof covering a stadium is a portion of a sphere. If the diameter of the floor of the stadium is 200 m and the roof is 75 m above the centre of the playing surface, what is the radius of the roof's shape?

The answer can be found by solving the following equation for ***r***, where ***h*** is the height of the roof, and ***c*** is the diameter of the stadium's base.

$$\sqrt{4h(2r-h)}=c$$

The solution for this problem will be shown later in the **Extensions** section of this topic.

What Lies Ahead

Throughout the topic you will learn to

1. solve and verify radical equations involving a single radical, and identify possible solutions of radical equations as being extraneous
2. apply simple radical equations to solve problems

Now that you know what to expect, turn the page to begin your study of solving and applying radical equations.

Activity 1

Solve and verify radical equations involving a single radical, and identify possible solutions of radical equations as being extraneous.

You may study this section by doing either **Part A** or **Part B** or both. **Part A** covers the activity through the print mode while **Part B** covers the objective with an audiotape. Whichever way you choose to study this activity, do the questions at the end of **Part B**.

Part A

Equations which have a variable as part of a radicand are known as radical equations.

$$\sqrt{y}=10 \qquad \sqrt{a}+3=5 \qquad \sqrt{z-2}=z+6$$

$$\sqrt{y-4}+6=y-3$$

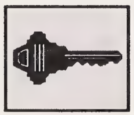
When you are asked to solve radical equations, the skills used to solve ordinary linear equations and common quadratic equations are applied. The following are suggested steps to take when solving radical equations.

- Begin by doing whatever is necessary to isolate the radical on either the left side or the right side of the equation.

- Continue by squaring both sides of the equation to eliminate the radical.

- Solve for the variable.

Once the solution is found, always verify the solution by checking whether the left and right sides are equal. Be sure to use the original equation for verification purposes. You will often find that all roots that result do not satisfy the conditions of the equation. Roots which do not satisfy the conditions of the equation are called **extraneous roots**. The extraneous roots, if they exist, should be clearly classified as being of this nature. Study the following examples to see how radical equations are solved and how extraneous roots, if they exist, are determined.



To check for an extraneous root, substitute that particular value in the left side (LS) and the right side (RS) of the original equation.

Solutions to equations are also called roots.

Recall: An example of a linear equation is as follows:

$$3x-7=13$$

An example of a quadratic equation is as follows:

$$3x^2-4x+7=0$$

Example 1

- Solve for x in the equation $\sqrt{x}=2$.

Solution:

$$\sqrt{x}=2$$
$$\left(\sqrt{x}\right)^2=(2)^2$$
$$x=4$$

Check:

$x=4$

LS	RS
$\sqrt{x}$	2
$\sqrt{4}$	2
2	2

LS = RS
(checks)

The solution is 4.

- Solve for x in the equation $\sqrt{x+3}-4=0$.

Solution:

$$\sqrt{x+3}-4=0$$
$$\sqrt{x+3}-4+4=0+4$$
$$\sqrt{x+3}=4$$
$$\left(\sqrt{x+3}\right)^2=(4)^2$$
$$x+3=16$$
$$x=16-3$$
$$x=13$$

Check:

$x=13$

LS	RS
$\sqrt{x+3}-4$	0
$\sqrt{13+3}-4$	0
$\sqrt{16}-4$	0
$4-4$	0
0	0

LS = RS
(checks)

The solution is 13.

To isolate the radical $\sqrt{x+3}$, add 4 to both sides of the equation.

Recall:

$$\left(\sqrt{2}\right)^2=2$$
$$\left(\sqrt{7}\right)^2=7$$
$$\left(\sqrt{a}\right)^2=a$$
$$\left(\sqrt{x}\right)^2=x$$
$$\left(\sqrt{x+1}\right)^2=x+1$$

- Solve for x in the equation $\sqrt{x+7}+x=13$.

Solution:

$$\sqrt{x+7}+x=13$$
$$\sqrt{x+7}+x-x=13-x$$
$$\sqrt{x+7}=13-x$$
$$\left(\sqrt{x+7}\right)^2=(13-x)^2$$
$$x+7=169-26x+x^2$$
$$x^2-26x+169=x+7$$
$$x^2-26x-x+169-7=x-x+7-7$$
$$x^2-27x+162=0 \qquad \text{(Factor the trinomial.)}$$
$$(x-9)(x-18)=0$$
$$x-9=0 \quad \text{or} \quad x-18=0$$
$$x=9 \qquad\qquad x=18$$

Check: $x=18$

LS	RS
$\sqrt{x+7}+x$	13
$\sqrt{18+7}+18$	13
$\sqrt{25}+18$	13
$5+18$	13
23	13

LS $\neq$ RS
(does not check)

$x=9$

LS	RS
$\sqrt{x+7}+x$	13
$\sqrt{9+7}+9$	13
$\sqrt{16}+9$	13
$4+9$	13
13	13

LS $=$ RS
(checks)

The solution is 9. The other root, 18, is an extraneous root since the LS and the RS of the check are not equal.

To isolate $\sqrt{x+7}$, subtract x from both sides of the equation.

Recall: Squaring a binomial is done as follows:

$$\begin{aligned}(13-x)^2 &= (13-x)(13-x)\\ &= 169-13x-13x+x^2\\ &= 169-26x+x^2\end{aligned}$$

- Solve for x in the equation $x+1=\sqrt{4x+197}-x$.

Solution:

$$x+1=\sqrt{4x+197}-x$$
$$x+x+1=\sqrt{4x+197}-x+x$$
$$2x+1=\sqrt{4x+197}$$
$$(2x+1)^2=\left(\sqrt{4x+197}\right)^2$$
$$4x^2+4x+1=4x+197$$
$$4x^2+4x-4x+1-197=4x-4x+197-197$$
$$4x^2-196=0$$
$$4\left(x^2-49\right)=0$$
$$4(x+7)(x-7)=0$$

$x+7=0$ or $x-7=0$

$x=-7$ $\quad x=7$

Check:

$x=-7$

LS	RS
$x+1$	$\sqrt{4x+197}-x$
$-7+1$	$\sqrt{4(-7)+197}-(-7)$
-6	$\sqrt{-28+197}+7$
-6	$\sqrt{169}+7$
-6	$13+7$
-6	20

LS $\neq$ RS

(does not check)

$x=7$

LS	RS
$x+1$	$\sqrt{4x+197}-x$
$7+1$	$\sqrt{4(7)+197}-7$
8	$\sqrt{28+197}-7$
8	$\sqrt{225}-7$
8	$15-7$
8	8

LS = RS

(checks)

The solution is 7. The other root, -7, is an extraneous root since the LS and the RS are not equal in the check.

Now do the questions which follow the audio activity.

Part B

Audio Activity

Solving radical equations can be difficult for some students. Listening to this audiotape may provide you with some additional ideas on solving radical equations. Insert the tape entitled Math 33 Unit 1 – Solving Radical Equations into your tape recorder and follow the instructions on the tape. After you have listened to the tape, complete the questions which follow.

1 Equations Involving Radicals

$$T = 2\pi\sqrt{\frac{l}{32}}$$

$$r = \frac{1}{2}\sqrt{\frac{s}{11}}$$

2 What Is a Radical Equation?

Is $5 = x + \sqrt{3}$ a radical equation?
No.

Is $\sqrt{3x-4} = 5$ a radical equation?
Yes.

A radical equation has a variable under the radical sign.

3 Solving Radical Equations with One Isolated Radical

$\sqrt{3x-5} = 4$	Square both sides.
$3x - 5 = 16$	Add 5 to both sides.
$3x = 21$	Divide by 3.
$x = 7$	

Check:

$x = 7$

LS	RS
$\sqrt{3x-5}$	4
$\sqrt{3(7)-5}$	4
$\sqrt{21-5}$	4
$\sqrt{16}$	4
4	4

LS = RS
(checks)

The solution of $\sqrt{3x-5} = 4$ is $x = 7$.

4 Solving Radical Equations When the Radical Is Not Isolated

$$
\begin{array}{rll}
\sqrt{y-1}+3=y & & \text{Isolate the radical.}\\
\sqrt{y-1}=y-3 & & \\
\left(\sqrt{y-1}\right)^2=(y-3)^2 & & \text{Square both sides.}\\
y-1=y^2-6y+9 & & \text{Collect like terms and move to the left side.}\\
y^2-7y+10=0 & & \\
(y-5)(y-2)=0 & & \text{Factor.}\\
y-5=0 \text{ or } y-2=0 & & \\
y=5 \qquad y=2 & &
\end{array}
$$

Check:

$y=5$

LS	RS
$\sqrt{y-1}+3$	y
$\sqrt{5-1}+3$	5
$\sqrt{4}+3$	5
$2+3$	5
5	5

LS = RS
(checks)

$y=2$

LS	RS
$\sqrt{y-1}+3$	y
$\sqrt{2-1}+3$	2
$\sqrt{1}+3$	2
$1+3$	2
4	2

LS $\neq$ RS
(does not check)

The solution is $y=5$.

5 Solving Radical Equations with Two Radicals

$$
\begin{array}{rl}
\sqrt{x+4}=4-\sqrt{x-4} & \text{Square both sides.}\\
x+4=16-8\sqrt{x-4}+x-4 & \text{Collect like terms.}\\
x+4=-8\sqrt{x-4}+x+12 & \\
-8=-8\sqrt{x-4} & \text{Square both sides again.}\\
64=64(x-4) & \text{Divide each side by 64.}\\
1=x-4 & \\
x=5 &
\end{array}
$$

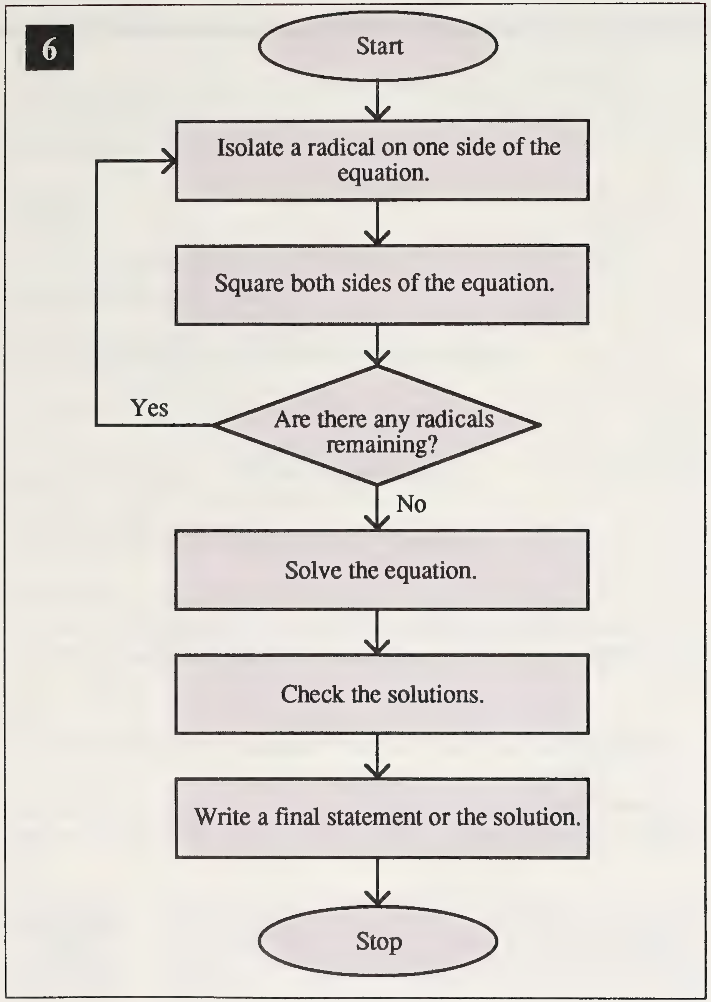
Check:

$x=5$

LS	RS
$\sqrt{x+4}$	$4-\sqrt{x-4}$
$\sqrt{5+4}$	$4-\sqrt{5-4}$
$\sqrt{9}$	$4-\sqrt{1}$
3	$4-1$
3	3

LS = RS
(checks)

The solution is $x=5$.



Solve the odd-numbered radical equations. Check your solutions and specify which roots are extraneous and which are not extraneous. If you need more practice, do the remaining questions.

1. $\sqrt{x}=4$	2. $3\sqrt{k}=15$
3. $\sqrt{y+5}=-1$	4. $\sqrt{x-4}=6$
5. $\sqrt{x+6}-3=1$	6. $\sqrt{2h+1}+4=3$
7. $\sqrt{x-1}-x+7=0$	8. $2\sqrt{x}=x-3$

For solutions to **Activity 1**, turn to the **Appendix, Topic 3**.

Activity 2

Apply simple radical equations to solve problems.

Radical equations are often used to describe actual situations. After the equation is derived, it is solved to find the information required.

No matter what type of problem is to be solved, use the following steps.

- Read the problem carefully to determine the given information and the required information. Choose a variable to represent the unknown.
- Organize the information into an equation.
- Solve the equation using all necessary skills.
- Check the solutions in the original equation.
- Use a closing statement to complete the solution.

Study the following examples.

Example 2

Natasha challenged her friend Benoit by presenting this problem: Find a number such that when its square root is multiplied by 3, and then 2 is subtracted from this product, the result is 4.

Solution:

Examine this equation to see how it states the information in symbolic form.

Let x represent the unknown number.

$$3\sqrt{x}-2=4$$
$$3\sqrt{x}-2+2=4+2$$
$$3\sqrt{x}=6$$
$$\left(3\sqrt{x}\right)^2=6^2$$
$$9x=36$$
$$\frac{9x}{9}=\frac{36}{9}$$
$$x=4$$

Check:

$x=4$

LS	RS
$3\sqrt{x}-2$	4
$3\sqrt{4}-2$	4
$3(2)-2$	4
$6-2$	4
4	4

LS = RS

(checks)

The number that Natasha wanted is 4.

Example 3

A sum of money is squared, then multiplied by 3, and finally increased by 4. When the square root of the resulting amount is taken, the result is 4. Find the amount of money.

Solution:

Let x represent the sum of money.

$$\sqrt{3x^2+4}=4$$

$$\left(\sqrt{3x^2+4}\right)^2=(4)^2$$

$$3x^2+4=16$$

$$3x^2+4-16=0$$

$$3x^2-12=0$$

$$3\left(x^2-4\right)=0$$

$$3(x-2)(x+2)=0$$

$$x-2=0 \quad \text{or} \quad x+2=0$$

$$x=2 \qquad\qquad x=-2$$

Check:

$x=2$

LS	RS
$\sqrt{3x^2+4}$	4
$\sqrt{3(2)^2+4}$	4
$\sqrt{3(4)+4}$	4
$\sqrt{12+4}$	4
$\sqrt{16}$	4
4	4

LS = RS
(checks)

Check:

$x=-2$

LS	RS
$\sqrt{3x^2+4}$	4
$\sqrt{3(-2)^2+4}$	4
$\sqrt{3(4)+4}$	4
$\sqrt{12+4}$	4
$\sqrt{16}$	4
4	4

LS = RS
(checks)

Even though the root $x=-2$ checks, it is an extraneous root since a sum of money cannot be negative.

The sum of money involved was $2.

Example 4

The sides of a rectangle are known to be $\sqrt{x+8}$ for the length and x for the width. If the length is two units longer than the width, find the dimensions of the rectangle.

Solution:

$$\sqrt{x+8}-x=2$$
$$\sqrt{x+8}=x+2$$
$$\left(\sqrt{x+8}\right)^2=(x+2)^2$$
$$x+8=x^2+4x+4$$
$$x^2+3x-4=0$$
$$(x+4)(x-1)=0$$
$$x+4=0 \quad \text{or} \quad x-1=0$$
$$x=-4 \qquad x=1$$

Check:

$x=-4$

LS	RS
$\sqrt{x+8}-x$	2
$\sqrt{-4+8}-(-4)$	2
$\sqrt{4}+4$	2
$2+4$	2
6	2

LS $\neq$ RS
(does not check)

The root is extraneous. Also, a measure cannot be negative.

Check:

$x=1$

LS	RS
$\sqrt{x+8}-x$	2
$\sqrt{1+8}-1$	2
$\sqrt{9}-1$	2
$3-1$	2
2	2

LS = RS
(checks)

The length of the rectangle is three units and the width is one unit.

Now it is time to put your understanding into practice.

Do problems 1, 3, and 5. If you need extra practice, do problems 2 and 4.

Use radical equations to solve each problem. Make sure to check your roots to eliminate extraneous roots.

1. When a number is increased by one and the square root of this sum is taken, the result is 3. Find the number.

2. The number of cars in a showroom is squared and 9 is added. When the square root of this value is found, the manager found that the result is the same as multiplying the number of cars by 2 and decreasing this product by 3. Find the number of cars in the showroom.

3. The heights of two bronze statues, in metres, are $\sqrt{2y+1}$ and $\frac{y}{2}$. Their combined height is 5 m. How high is each statue?

4. The square root of a number is multiplied by 3, and 7 is subtracted from the product. The result is 8. What is the number?

5. The winner of a lottery prize had to solve the following skill-testing problem:
A number is doubled and increased by 1. When the square root of this value is tripled, the result is 15. Find the number.

For solutions to **Activity 2**, turn to the **Appendix, Topic 3**.

Remember: The steps for problem solving are as follows:

- Read the problem carefully and choose a variable to represent the unknown.
- Organize the information into an equation.
- Solve the equation.
- Check the solutions.
- Give the answer in a closing statement.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

For solving radical equations, carefully review the following basic skills.

- When solving equations in general, you must isolate the variable. For example, in solving $2x-3=7$, you would add 3 to both sides of the equation and then divide by 2.

$$\begin{aligned} 2x-3&=7\\ 2x-3+3&=7+3\\ 2x&=10\\ \frac{2x}{2}&=\frac{10}{2}\\ x&=5 \end{aligned}$$

When solving radical equations, you must first isolate the radical. For example, in solving $2\sqrt{x}+3=13$, you would write the following:

$$\begin{aligned} 2\sqrt{x}+3&=13\\ 2\sqrt{x}+3-3&=13-3\\ 2\sqrt{x}&=10\\ \frac{2\sqrt{x}}{2}&=\frac{10}{2}\\ \sqrt{x}&=5 \end{aligned}$$

To isolate the radical, you do the operations that are necessary to get the radical by itself on either the left side or the right side of the equation. The following are some of the situations which will occur.

$$\begin{aligned} \sqrt{x}+7&=14\\ \sqrt{x}+7-7&=14-7\\ \sqrt{x}&=7 \end{aligned}$$

The inverse of addition is subtraction, so 7 is subtracted from both sides.

$$\begin{aligned} \sqrt{2x+1}-3&=17\\ \sqrt{2x+1}-3+3&=17+3\\ \sqrt{2x+1}&=20 \end{aligned}$$

The inverse of subtraction is addition, so 3 is added to both sides.

$$3\left(\sqrt{7x+4}\right)=12$$

$$\frac{3\left(\sqrt{7x+4}\right)}{3}=\frac{12}{3}$$

$$\sqrt{7x+4}=4$$

The inverse of multiplication is division, so both sides are divided by 3.

$$\frac{\sqrt{5x}}{3}=2$$

$$\frac{3\left(\sqrt{5x}\right)}{3}=2\times 3$$

$$\sqrt{5x}=6$$

The inverse of division is multiplication, so both sides are multiplied by 3.

Often you have to apply more than one operation as the following illustrates.

$$\frac{\sqrt{10x+2}}{5}+3=9$$

$$\frac{\sqrt{10x+2}}{5}+3-3=9-3$$

$$\frac{\sqrt{10x+2}}{5}=6$$

$$5\left(\frac{\sqrt{10x+2}}{5}\right)=5\times 6$$

$$\sqrt{10x+2}=30$$

OR

$$\frac{\sqrt{10x+2}}{5}+3=9$$

$$5\left(\frac{\sqrt{10x+2}}{5}\right)+5(3)=5(9)$$

$$\sqrt{10x+2}+15=45$$

$$\sqrt{10x+2}+15-15=45-15$$

$$\sqrt{10x+2}=30$$

- Once the radical is isolated, square both sides to eliminate the radical sign. Keep in mind that squaring any second-order radical will eliminate the radical sign. The following situations show what happens. (Complete detailed steps are used.)

You may choose to solve the equation $\frac{\sqrt{10x+2}}{5}+3=9$ in either order as shown. Either order will lead to the same result. Study both methods.

Note: In the second part, all three terms are multiplied by 5.

For $\sqrt{5}$,

$$\begin{aligned}\left(\sqrt{5}\right)^2 &= \sqrt{5} \times \sqrt{5}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

For $2\sqrt{3}$,

$$\begin{aligned}\left(2\sqrt{3}\right)^2 &= 2\sqrt{3} \times 2\sqrt{3}\\ &= 4\sqrt{9}\\ &= 4 \times 3\\ &= 12\end{aligned}$$

For $\sqrt{3x}$,

$$\begin{aligned}\left(\sqrt{3x}\right)^2 &= \sqrt{3x} \times \sqrt{3x}\\ &= \sqrt{9x^2}\\ &= 3x\end{aligned}$$

For $\sqrt{2x-1}$,

$$\begin{aligned}\left(\sqrt{2x-1}\right)^2 &= \sqrt{2x-1} \times \sqrt{2x-1}\\ &= \sqrt{(2x-1)^2}\\ &= 2x-1\end{aligned}$$

All the detail is not really necessary because the square root sign simply disappears. Here is another example.

$$\left(\sqrt{3}\right)^2 = 3$$

$$\begin{aligned}\left(3\sqrt{5}\right)^2 &= 9 \times 5\\ &= 45\end{aligned}$$

$$\left(\sqrt{3y}\right)^2 = 3y$$

$$\left(\sqrt{5x+4}\right)^2 = 5x+4$$

As soon as the radical sign disappears, proceed to isolate and solve for the variable.

- All roots must be verified since some may not satisfy the conditions of the equation. Those that do not check are called extraneous roots and should be excluded. Always start the check with the original equation.

Take time to apply these concepts to the following questions.

Do questions 1b, 1d, 1f, 1h, and 2. If you need more practice, do the other questions.

1. Solve each of the following. Check all roots.

   a. $\sqrt{x}=6$

   b. $\sqrt{m}=3$

   c. $3\sqrt{x}-2=4$

   d. $3\sqrt{2x+1}=15$

   e. $\sqrt{5x-1}-1=x$

   f. $\sqrt{2n^2-n+4}-n=2$

   g. $\sqrt{5p+4}=5-2p$

   h. $4+2\sqrt{5x-3}=12$

2. The square root of the difference of two times a number and 3 is added to 3. The sum is the original number. Find this number.

For solutions to **Extra Help**, turn to the **Appendix, Topic 3**.

Extensions

To begin this section, you will be shown how to solve for r in the problem which was given in the **Introduction** to **Topic 3**.

Solve for r when $c = 200$ and $h = 75$.

$$\begin{aligned}
\sqrt{4h(2r-h)} &= c \\
\sqrt{4(75)(2r-75)} &= 200 \\
\left(\sqrt{300(2r-75)}\right)^2 &= (200)^2 \\
300(2r-75) &= 40\,000 \\
600r-22\,500 &= 40\,000 \\
600r-22\,500+22\,500 &= 40\,000+22\,500 \\
600r &= 62\,500 \\
&= \frac{62\,500}{600} \\
r &= \frac{625}{6} \\
r &\doteq 104.17
\end{aligned}$$

The radius of the curved roof is approximately 104 m.

The previous equation and all the others in this topic contain only one radical. However, there is no limit to the number of radicals that can be placed in a radical equation.

When you study the examples provided, try to see what is involved in solving these equations as compared to the solving of equations which contain one radical. You will see that the squaring process is used more than once in most cases. The reason for this is that all radicals must be eliminated before applying other skills to solve for the variable. Once the solutions are found, they must be verified to determine whether they satisfy the conditions of the equation or are extraneous.

Example 5

Solve $\sqrt{3x}+1=\sqrt{5x+1}$.

Solution:

$$\left(\sqrt{3x}+1\right)^2=\left(\sqrt{5x+1}\right)^2$$
$$3x+2\sqrt{3x}+1=5x+1$$
$$2\sqrt{3x}=5x-3x+1-1$$
$$2\sqrt{3x}=2x$$
$$\left(2\sqrt{3x}\right)^2=(2x)^2$$
$$4(3x)=4x^2$$
$$12x=4x^2$$
$$4x^2-12x=0$$
$$4x(x-3)=0$$
$$4x=0 \quad \text{or} \quad x-3=0$$
$$x=0 \qquad\qquad x=3$$

Check:

$x=0$

LS	RS
$\sqrt{3x}+1$	$\sqrt{5x+1}$
$\sqrt{3(0)}+1$	$\sqrt{5(0)+1}$
$\sqrt{0}+1$	$\sqrt{0+1}$
$0+1$	$\sqrt{1}$
1	1

LS = RS
(checks)

Check:

$x=3$

LS	RS
$\sqrt{3x}+1$	$\sqrt{5x+1}$
$\sqrt{3(3)}+1$	$\sqrt{5(3)+1}$
$\sqrt{9}+1$	$\sqrt{15+1}$
$3+1$	$\sqrt{16}$
4	4

LS = RS
(checks)

The solutions are 0 and 3.

Be careful when squaring $\left(\sqrt{3x}+1\right)$. Apply the FOIL rule.

$$\left(\sqrt{3x}+1\right)^2$$
$$=\left(\sqrt{3x}+1\right)\left(\sqrt{3x}+1\right)$$
$$=\left(\sqrt{3x}\right)^2+\sqrt{3x}+\sqrt{3x}+1$$
$$=3x+2\sqrt{3x}+1$$

Example 6

Solve $\sqrt{3x+6}-\sqrt{x+6}=2$.

Solution:

Before squaring the equation, move one radical to the other side so you have a radical on each side. Then square both sides of the equation.

$$\begin{aligned}
\sqrt{3x+6}-\sqrt{x+6}&=2\\
\sqrt{3x+6}&=2+\sqrt{x+6}\\
\left(\sqrt{3x+6}\right)^2&=\left(2+\sqrt{x+6}\right)^2\\
3x+6&=4+4\sqrt{x+6}+x+6\\
3x-x+6-10&=4\sqrt{x+6}\\
2x-4&=4\sqrt{x+6}\\
(2x-4)^2&=\left(4\sqrt{x+6}\right)^2\\
4x^2-16x+16&=16(x+6)\\
4x^2-16x+16&=16x+96\\
4x^2-16x-16x+16-96&=0\\
4x^2-32x-80&=0\\
\frac{4\left(x^2-8x-20\right)}{4}&=\frac{0}{4}\\
x^2-8x-20&=0\\
(x-10)(x+2)&=0
\end{aligned}$$

$$x-10=0 \quad \text{or} \quad x+2=0$$
$$x=10 \qquad\qquad x=-2$$

Check:

$x=10$

LS	RS
$\sqrt{3x+6}-\sqrt{x+6}$	2
$\sqrt{3(10)+6}-\sqrt{10+6}$	2
$\sqrt{30+6}-\sqrt{16}$	2
$\sqrt{36}-\sqrt{16}$	2
$6-4$	2
2	2

LS = RS
(checks)

Check:

$x=-2$

LS	RS
$\sqrt{3x+6}-\sqrt{x+6}$	2
$\sqrt{3(-2)+6}-\sqrt{-2+6}$	2
$\sqrt{-6+6}-\sqrt{4}$	2
$\sqrt{0}-2$	2
$0-2$	2
-2	2

LS $\neq$ RS
(does not check)

The solution is 10. The root $x=-2$ is an extraneous root.

Example 7

Solve $\dfrac{1}{\sqrt{x-2}} = \dfrac{\sqrt{x-2}}{\sqrt{2x+4}}$.

Solution:

Find the cross products since you are working with ratios.

$$\begin{aligned}
\frac{1}{\sqrt{x-2}} &= \frac{\sqrt{x-2}}{\sqrt{2x+4}} \\
\left(\sqrt{x-2}\right)\left(\sqrt{x-2}\right) &= \sqrt{2x+4} \\
(x-2) &= \sqrt{2x+4} \\
(x-2)^2 &= \left(\sqrt{2x+4}\right)^2 \\
x^2 - 4x + 4 &= 2x + 4 \\
x^2 - 6x &= 0 \\
x(x-6) &= 0 \\
x = 0 \quad \text{or} \quad x - 6 &= 0 \\
x &= 6
\end{aligned}$$

Check:

$x = 0$

LS	RS
$\frac{1}{\sqrt{x-2}}$	$\frac{\sqrt{x-2}}{\sqrt{2x+4}}$
$\frac{1}{\sqrt{0-2}}$	$\frac{\sqrt{0-2}}{\sqrt{2(0)+4}}$
$\frac{1}{\sqrt{-2}}$	$\frac{\sqrt{-2}}{\sqrt{4}}$

Since $\sqrt{-2}$ is undefined, the solution is not 0.

Check:

$x = 6$

LS	RS
$\frac{1}{\sqrt{x-2}}$	$\frac{\sqrt{x-2}}{\sqrt{2x+4}}$
$\frac{1}{\sqrt{6-2}}$	$\frac{\sqrt{6-2}}{\sqrt{2(6)+4}}$
$\frac{1}{\sqrt{4}}$	$\frac{\sqrt{4}}{\sqrt{12+4}}$
$\frac{1}{2}$	$\frac{2}{\sqrt{16}}$
$\frac{1}{2}$	$\frac{2}{4}$
$\frac{1}{2}$	$\frac{1}{2}$

LS = RS
(checks)

The solution is 6.

Example 8

Find the number for which the following properties exist:

The square root of the sum of five times a number and 34 is diminished by the square root of the sum of five times the number and 6. The difference is 2.

Solution:

Let x be the required number.

$$
\begin{aligned}
\sqrt{5x+34}-\sqrt{5x+6} &= 2 \\
\sqrt{5x+34} &= 2+\sqrt{5x+6} \\
\left(\sqrt{5x+34}\right)^2 &= \left(2+\sqrt{5x+6}\right)^2 \\
5x+34 &= 4+4\sqrt{5x+6}+5x+6 \\
5x+34 &= 10+5x+4\sqrt{5x+6} \\
4\sqrt{5x+6} &= 24 \\
\left(4\sqrt{5x+6}\right)^2 &= (24)^2 \\
16(5x+6) &= 576 \\
80x+96 &= 576 \\
80x &= 480 \\
x &= 6
\end{aligned}
$$

Check:

$x = 6$

LS	RS
$\sqrt{5x+34}-\sqrt{5x+6}$	2
$\sqrt{5(6)+34}-\sqrt{5(6)+6}$	2
$\sqrt{30+34}-\sqrt{30+6}$	2
$\sqrt{64}-\sqrt{36}$	2
$8-6$	2
2	2

LS = RS
(checks)

The required number is 6.

Now it is your turn to try some similar questions.

Do questions 1a, 1c, 1e, and 2. If you want more practice, do the other questions.

1. Solve each of the following equations. Check all solutions to determine and reject extraneous roots.

   a. $\sqrt{2y+5}-\sqrt{y-2}=3$

   b. $\sqrt{4-6x}-1=\sqrt{-5x-1}$

   c. $\sqrt{3-x}+\sqrt{2x+3}=3$

   d. $\sqrt{x+9}-\sqrt{x-6}=3$

   e. $\dfrac{1}{\sqrt{x+1}}=\dfrac{\sqrt{2x+3}}{2x}$

2. Two properties exist such that when they are added, the result is 5. These properties are as follows:

   - the square root of one more than three times a number
   - the square root of four less than the number

   Find the number.

For the solutions to **Extensions**, turn to the **Appendix, Topic 3**.

Unit Summary

What You Have Learned

Having completed this unit, you should be able to

- change mixed radicals to entire radicals and vice versa
- write a radical in its simplest form
- add, subtract, multiply, and divide radicals
- recognize conjugates
- find the product of conjugates
- simplify a quotient by rationalizing the denominator
- solve radical equations
- recognize the necessity of verifying solutions to radical equations
- solve practical problems involving the use of radical equations

All of the previous skills will help you work with exact values when working with numbers that are not perfect squares. For example, in $\sqrt{5} \doteq 2.236$ the radical $\sqrt{5}$ is an exact value while 2.236 is an approximation.

You are now ready to complete the **Unit Assignment**.

Appendix

Solutions

Review

Topic 1 Changing the Form of a Radical and Adding and Subtracting Radicals

Topic 2 Multiplying and Dividing Radicals

Topic 3 Solving and Applying Radical Equations

Review

1. a. $6^3 = 6 \times 6 \times 6$
$= 216$

b. $5^{-2} = \frac{1}{5^2}$
$= \frac{1}{5 \times 5}$
$= \frac{1}{25}$

c. $(-2)^4 = (-2)(-2)(-2)(-2)$
$= 16$

d. $7^0 = 1$

2. a. $t^4 \times t^5 = t^{4+5}$
$= t^9$

b. $k^8 \div k^3 = k^{8-3}$
$= k^5$

c. $(a^4)^3 = a^4 \times a^4 \times a^4$
$= a^{4+4+4}$
$= a^{12}$

or $(a^4)^3 = a^{4 \times 3}$
$= a^{12}$

d. $\left(\frac{p^3}{q}\right)^2 = \frac{p^3}{q} \times \frac{p^3}{q}$
$= \frac{p^{3+3}}{q^{1+1}}$
$= \frac{p^6}{q^2}$

or $\left(\frac{p^3}{q}\right)^2 = \frac{p^{3 \times 2}}{q^{1 \times 2}}$
$= \frac{p^6}{q^2}$

e. $(m^3 n)^2 = m^3 n \times m^3 n$
$= m^{3+3} n^{1+1}$
$= m^6 n^2$

or $(m^3 n)^2 = m^{3 \times 2} n^{1 \times 2}$
$= m^6 n^2$

f. $(p^5)^{-2} = \frac{1}{(p^5)^2}$
$= \frac{1}{(p^5) \times (p^5)}$
$= \frac{1}{p^{5+5}}$
$= \frac{1}{p^{10}}$

or $(p^5)^{-2} = p^{5 \times -2}$
$= p^{-10}$
$= \frac{1}{p^{10}}$

g. $t^{-7} \times t^{5} = t^{-7+5}$

$= t^{-2}$

$= \dfrac{1}{t^{2}}$

h. $m^{-1} \div m^{-3} = m^{-1-(-3)}$

$= m^{-1+3}$

$= m^{2}$

i. $\dfrac{(m^{-3}n^{-5})(m^{2}n^{4})}{(m^{2}n)^{-2}} = \dfrac{m^{-3+2}n^{-5+4}}{m^{2\times-2}n^{1\times-2}}$

$= \dfrac{m^{-1}n^{-1}}{m^{-4}n^{-2}}$

$= m^{-1-(-4)}n^{-1-(-2)}$

$= m^{-1+4}n^{-1+2}$

$= m^{3}n$

3. a. $\sqrt[3]{4} = 4^{\frac{1}{3}}$

b. $(\sqrt{a})^{3} = a^{\frac{3}{2}}$

c. $\left(\dfrac{1}{\sqrt[3]{y}}\right)^{2} = \dfrac{1}{y^{\frac{2}{3}}}$

4. a. $(2)^{\frac{1}{2}} = \sqrt{2}$

b. $h^{\frac{2}{3}} = (\sqrt[3]{h})^{2}$ or $h^{\frac{2}{3}} = \sqrt[3]{h^{2}}$

c. $x^{-\frac{1}{3}} = \dfrac{1}{\sqrt[3]{x}}$

d. $d^{-\frac{2}{5}} = \dfrac{1}{(\sqrt[5]{d})^{2}}$ or $d^{-\frac{2}{5}} = \dfrac{1}{\sqrt[5]{d^{2}}}$

5. a. Since $(-3)(-3)(-3) = -27$, $\sqrt[3]{-27} = -3$.

b. $(\sqrt{25})^{3} = 5^{3}$

$= 5 \times 5 \times 5$

$= 125$

c. $\sqrt[3]{(-1)^{4}} = \sqrt[3]{(-1)(-1)(-1)(-1)}$

$= \sqrt[3]{1}$

$= 1$

d. $\left(\sqrt{9}\right)^0 = 3^0$

$= 1$

e. $49^{\frac{3}{2}} = \left(\sqrt{49}\right)^3$

$= 7^3$

$= 343$

f. Since $(-4)(-4)(-4) = -64$, $(-64)^{\frac{1}{3}} = \sqrt[3]{-64} = -4$.

g. $25^{-\frac{1}{2}} = \dfrac{1}{25^{\frac{1}{2}}}$

$= \dfrac{1}{\sqrt{25}}$

$= \dfrac{1}{5}$

h. $(-8)^{\frac{2}{3}} = \left(\sqrt[3]{(-8)}\right)^2$

$= (-2)^2$

$= 4$

6. a. $\dfrac{5}{6} + \dfrac{7}{18} = \dfrac{15}{18} + \dfrac{7}{18}$

$= \dfrac{22}{18}$

$= 1\dfrac{4}{18}$

$= 1\dfrac{2}{9}$

b. $1\dfrac{1}{3} + 7\dfrac{1}{2} + 4\dfrac{2}{9} = \dfrac{4}{3} + \dfrac{15}{2} + \dfrac{38}{9}$

$= \dfrac{24}{18} + \dfrac{135}{18} + \dfrac{76}{18}$

$= \dfrac{235}{18}$

$= 13\dfrac{1}{18}$

c. $\dfrac{11}{12} - \dfrac{4}{9} = \dfrac{33}{36} - \dfrac{16}{36}$

$= \dfrac{17}{36}$

d. $5\dfrac{1}{3} - 2\dfrac{7}{10} = 5\dfrac{10}{30} - 2\dfrac{21}{30}$

$= 4\dfrac{40}{30} - 2\dfrac{21}{30}$

$= 2\dfrac{19}{30}$

7. a. like radicals

   b. unlike radicals

   c. like radicals

Exploring Topic 1

Activity 1

Change entire radicals to mixed radicals.

1. a. $\sqrt{20} = \sqrt{4 \times 5}$
   $= \sqrt{4} \times \sqrt{5}$
   $= 2\sqrt{5}$

   b. $\sqrt{300} = \sqrt{100 \times 3}$
   $= \sqrt{100} \times \sqrt{3}$
   $= 10\sqrt{3}$

   c. $\sqrt[3]{72} = \sqrt[3]{8 \times 9}$
   $= \sqrt[3]{8} \times \sqrt[3]{9}$
   $= 2\sqrt[3]{9}$

2. a. $\sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

   b. $\sqrt[3]{27} \times \sqrt[3]{9} = 3\sqrt[3]{9}$

   c. $\sqrt{36} \times \sqrt{5} = 6\sqrt{5}$

3. a. $\sqrt{400} = \sqrt{20^2}$
   $= 20$

   b. $\sqrt{225} = \sqrt{15^2}$
   $= 15$

   c. $\sqrt[3]{625} = \sqrt[3]{5^3 \times 5}$
   $= 5\sqrt[3]{5}$

4. a. $\sqrt{48} = \sqrt{16 \times 3}$
   $= \sqrt{16} \times \sqrt{3}$
   $= 4\sqrt{3}$

   b. $\sqrt{63} = \sqrt{9 \times 7}$
   $= \sqrt{9} \times \sqrt{7}$
   $= 3\sqrt{7}$

c. $\sqrt{1000} = \sqrt{100 \times 10}$
$= \sqrt{100} \times \sqrt{10}$
$= 10\sqrt{10}$

d. $\sqrt[3]{2000} = \sqrt[3]{1000 \times 2}$
$= \sqrt[3]{1000} \times \sqrt[3]{2}$
$= 10\sqrt[3]{2}$

5. a. $\sqrt{240} = \sqrt{16 \times 15}$
$= \sqrt{16} \times \sqrt{15}$
$= 4\sqrt{15}$

The length of each side is $4\sqrt{15}$ m.

b. $4\sqrt{15} \doteq 4 \times 3.873$
$\doteq 15.492$
$\doteq 15.5$

The length of each side is about 15.5 m.

Activity 2

Change mixed radicals to entire radicals.

1. a. $3\sqrt{5} = \sqrt{3^2} \times \sqrt{5}$
$= \sqrt{9} \times \sqrt{5}$
$= \sqrt{45}$

b. $4\sqrt{3} = \sqrt{4^2} \times \sqrt{3}$
$= \sqrt{16} \times \sqrt{3}$
$= \sqrt{48}$

c. $7\sqrt[3]{2} = \sqrt[3]{7^3} \times \sqrt[3]{2}$
$= \sqrt[3]{343} \times \sqrt[3]{2}$
$= \sqrt[3]{686}$

2. a. $\sqrt{2} \times \sqrt{5} = \sqrt{10}$

b. $\sqrt{6} \times \sqrt{3} = \sqrt{18}$

c. $\sqrt[3]{11} \times \sqrt[3]{7} = \sqrt[3]{77}$

Activity 3

Add and subtract radicals.

1. a. $6\sqrt{7}+5\sqrt{7}=(6+5)\sqrt{7}$
$$=11\sqrt{7}$$

b. $-8\sqrt{3}+12\sqrt{3}=(-8+12)\sqrt{3}$
$$=4\sqrt{3}$$

c. $-5\sqrt{22}-8\sqrt{22}=(-5-8)\sqrt{22}$
$$=-13\sqrt{22}$$

2. a. $7\sqrt{5}+9-2\sqrt{5}-4=7\sqrt{5}-2\sqrt{5}+9-4$
$$=(7-2)\sqrt{5}+5$$
$$=5\sqrt{5}+5$$

b. $-6\sqrt{10}+20-\left(4\sqrt{10}+11\right)=-6\sqrt{10}+20-4\sqrt{10}-11$
$$=-6\sqrt{10}-4\sqrt{10}+20-11$$
$$=(-6-4)\sqrt{10}+9$$
$$=-10\sqrt{10}+9 \quad \text{or} \quad 9-10\sqrt{10}$$

3. a. Change $\sqrt{48}$ and $\sqrt{75}$ to mixed radicals.

b. $\sqrt{48}+\sqrt{75}=\sqrt{16\times 3}+\sqrt{25\times 3}$
$$=4\sqrt{3}+5\sqrt{3}$$
$$=(4+5)\sqrt{3}$$
$$=9\sqrt{3}$$

4. a. $6\sqrt{2}+\sqrt{32}=6\sqrt{2}+\sqrt{16\times 2}$
$$=6\sqrt{2}+4\sqrt{2}$$
$$=(6+4)\sqrt{2}$$
$$=10\sqrt{2}$$

b. $5\sqrt{2}-\sqrt{98}=5\sqrt{2}-\sqrt{49\times 2}$
$$=5\sqrt{2}-7\sqrt{2}$$
$$=(5-7)\sqrt{2}$$
$$=-2\sqrt{2}$$

c. $\sqrt{96}-\sqrt{24}=\sqrt{16\times 6}-\sqrt{4\times 6}$
$$=4\sqrt{6}-2\sqrt{6}$$
$$=(4-2)\sqrt{6}$$
$$=2\sqrt{6}$$

5. a. $$\begin{aligned}\sqrt{48}+\sqrt{50}+6\sqrt{2}+\left(10\sqrt{2}-4\sqrt{3}\right)+\sqrt{12} &= \sqrt{16\times 3}+\sqrt{25\times 2}+6\sqrt{2}+10\sqrt{2}-4\sqrt{3}+\sqrt{4\times 3}\\ &= 4\sqrt{3}+5\sqrt{2}+6\sqrt{2}+10\sqrt{2}-4\sqrt{3}+2\sqrt{3}\\ &= 4\sqrt{3}-4\sqrt{3}+2\sqrt{3}+5\sqrt{2}+6\sqrt{2}+10\sqrt{2}\\ &= (4-4+2)\sqrt{3}+(5+6+10)\sqrt{2}\\ &= 2\sqrt{3}+21\sqrt{2}\end{aligned}$$

The distance around this figure is $\left(2\sqrt{3}+21\sqrt{2}\right)$ m.

b. $$\begin{aligned}2\sqrt{3}+21\sqrt{2} &\doteq 2\times 1.732+21\times 1.414\\ &\doteq 3.464+29.698\\ &\doteq 33.162\\ &\doteq 33.16\end{aligned}$$

The distance around this figure is about 33.16 m.

6. a. $$\begin{aligned}\sqrt{5}+\sqrt{20}-2\sqrt{5} &= \sqrt{5}+\sqrt{4\times 5}-2\sqrt{5}\\ &= \sqrt{5}+2\sqrt{5}-2\sqrt{5}\\ &= (1+2-2)\sqrt{5}\\ &= 1\sqrt{5}\\ &= \sqrt{5}\end{aligned}$$

b. $$\begin{aligned}\sqrt{108}-\sqrt{48}+\sqrt{300} &= \sqrt{36\times 3}-\sqrt{16\times 3}+\sqrt{100\times 3}\\ &= 6\sqrt{3}-4\sqrt{3}+10\sqrt{3}\\ &= (6-4+10)\sqrt{3}\\ &= 12\sqrt{3}\end{aligned}$$

7. a. $\sqrt{288}+\sqrt{800}+\sqrt{450}+\sqrt{162}=\sqrt{144\times 2}+\sqrt{400\times 2}+\sqrt{225\times 2}+\sqrt{81\times 2}$

$$=12\sqrt{2}+20\sqrt{2}+15\sqrt{2}+9\sqrt{2}$$
$$=(12+20+15+9)\sqrt{2}$$
$$=56\sqrt{2}$$

The distance between C and D is $56\sqrt{2}$ m.

b. $56\sqrt{2} \doteq 56\times 1.414\,213\,562$

$$\doteq 79.196$$
$$\doteq 79 \text{ m}$$

The distance between C and D is approximately 79 m.

Extra Help

1. a. $\sqrt{56}=\sqrt{2\times 2\times 2\times 7}$

$$=\sqrt{2^2\times 2\times 7}$$
$$=2\sqrt{14}$$

b. $\sqrt{63}=\sqrt{3\times 3\times 7}$

$$=\sqrt{3^2\times 7}$$
$$=3\sqrt{7}$$

c. $\sqrt{200} = \sqrt{2\times 2\times 2\times 5\times 5}$
$= \sqrt{2^2 \times 2 \times 5^2}$
$= 2\times 5\sqrt{2}$
$= 10\sqrt{2}$

d. $\sqrt{180} = \sqrt{2\times 2\times 3\times 3\times 5}$
$= \sqrt{2^2 \times 3^2 \times 5}$
$= 2\times 3\sqrt{5}$
$= 6\sqrt{5}$

e. $\sqrt{3564} = \sqrt{2\times 2\times 3\times 3\times 3\times 3\times 11}$
$= \sqrt{2^2 \times 3^2 \times 3^2 \times 11}$
$= 2\times 3\times 3\sqrt{11}$
$= 18\sqrt{11}$

f. $\sqrt{12\,600} = \sqrt{2\times 2\times 2\times 3\times 3\times 5\times 5\times 7}$
$= \sqrt{2^2 \times 2\times 3^2 \times 5^2 \times 7}$
$= 2\times 3\times 5\sqrt{2\times 7}$
$= 30\sqrt{14}$

2. a. $6\sqrt{5} = \sqrt{6\times 6\times 5}$
$= \sqrt{180}$

b. $11\sqrt{3} = \sqrt{11\times 11\times 3}$
$= \sqrt{363}$

c. $5\sqrt{10} = \sqrt{5\times 5\times 10}$
$= \sqrt{250}$

d. $9\sqrt{2} = \sqrt{9\times 9\times 2}$
$= \sqrt{162}$

e. $15\sqrt{3} = \sqrt{15\times 15\times 3}$
$= \sqrt{675}$

f. $2\sqrt{7} + 4\sqrt{7} = 6\sqrt{7}$
$= \sqrt{6\times 6\times 7}$
$= \sqrt{252}$

3. a. $\sqrt{60} + 2\sqrt{15} - 4\sqrt{15} = \sqrt{4\times 15} + 2\sqrt{15} - 4\sqrt{15}$
$= 2\sqrt{15} + 2\sqrt{15} - 4\sqrt{15}$
$= (2+2-4)\sqrt{15}$
$= 0\sqrt{15}$
$= 0$

b. $$\begin{aligned} 7-2\sqrt{3}+\sqrt{27}+9-5\sqrt{3}+\sqrt{48} &= 7-2\sqrt{3}+\sqrt{9\times 3}+9-5\sqrt{3}+\sqrt{16\times 3} \\ &= 7-2\sqrt{3}+3\sqrt{3}+9-5\sqrt{3}+4\sqrt{3} \\ &= 7+9-2\sqrt{3}+3\sqrt{3}-5\sqrt{3}+4\sqrt{3} \\ &= 16+(-2+3-5+4)\sqrt{3} \\ &= 16+0\sqrt{3} \\ &= 16+0 \\ &= 16 \end{aligned}$$

c. $$\begin{aligned} -3\sqrt{5}+\sqrt{180}-4\sqrt{5}+\sqrt{125} &= -3\sqrt{5}+\sqrt{36\times 5}-4\sqrt{5}+\sqrt{25\times 5} \\ &= -3\sqrt{5}+6\sqrt{5}-4\sqrt{5}+5\sqrt{5} \\ &= (-3+6-4+5)\sqrt{5} \\ &= 4\sqrt{5} \end{aligned}$$

Extensions

1. a. $$\begin{aligned} \sqrt{36a^2b^3c^4} &= \sqrt{6\times 6\times a\times a\times b\times b\times b\times c\times c\times c\times c} \\ &= \sqrt{6^2a^2b^2c^2c^2b} \\ &= 6\left|abc^2\right|\sqrt{b} \\ &= 6c^2\,|ab|\sqrt{b} \end{aligned}$$

b. $\sqrt{\frac{9}{16}x^5y^2} = \sqrt{\frac{3}{4}\times\frac{3}{4}\times x\times x\times x\times x\times x\times y\times y}$

$= \sqrt{\left(\frac{3}{4}\right)^2 x^2x^2y^2x}$

$= \frac{3}{4}\left|x^2y\right|\sqrt{x}$

$= \frac{3}{4}x^2\ |y|\sqrt{x}$

c. $\sqrt{0.16c^6d^3} = \sqrt{0.4\times0.4\times c\times c\times c\times c\times c\times c\times d\times d\times d}$

$= 0.4\left|c^3d\right|\sqrt{d}$

2. a. $5xyz\sqrt{2x} = \sqrt{5\times5\times2\times x\times x\times x\times y\times y\times z\times z}$

$= \sqrt{50x^3y^2z^2}$

b. $\frac{2}{5}a^2b\sqrt{10c} = \sqrt{\frac{2}{5}\times\frac{2}{5}\times10\times a^2\times a^2\times b\times b\times c}$

$= \sqrt{\frac{4}{25}\times10\times a^4b^2c}$

$= \sqrt{\frac{8}{5}a^4b^2c}$

c. $1.3c^3\sqrt{6d} = \sqrt{1.3\times1.3\times6\times c^3\times c^3\times d}$

$= \sqrt{1.69\times6\times c^6d}$

$= \sqrt{10.14c^6d}$

3. a. $4cd\sqrt{2f}+3cd\sqrt{2f}-5cd\sqrt{2f}-9cd\sqrt{2f}=(4cd+3cd-5cd-9cd)\sqrt{2f}$

$$=-7cd\sqrt{2f}$$

b. $-\sqrt{0.18x^3y}+\sqrt{5.12x^3y}-\sqrt{18x^3y}+\sqrt{32x^3y}$

$$=-\sqrt{0.09\times 2\times x^2\times x\times y}+\sqrt{2.56\times 2\times x^2\times x\times y}-\sqrt{9\times 2\times x^2\times x\times y}+\sqrt{16\times 2\times x^2\times x\times y}$$

$$=-0.3|x|\sqrt{2xy}+1.6|x|\sqrt{2xy}-3|x|\sqrt{2xy}+4|x|\sqrt{2xy}$$

$$=2.3|x|\sqrt{2xy}$$

Exploring Topic 2

Activity 1

Multiply radicals.

1. $\sqrt{3}\times\sqrt{6}=\sqrt{3\times 6}$

$$=\sqrt{18}$$

$$=\sqrt{9\times 2}$$

$$=\sqrt{9}\times\sqrt{2}$$

$$=3\sqrt{2}$$

2. $\sqrt{8}\times\sqrt{6}=\sqrt{8\times 6}$
$=\sqrt{48}$
$=\sqrt{16\times 3}$
$=\sqrt{16}\times\sqrt{3}$
$=4\sqrt{3}$

3. $4\sqrt{5}\times\sqrt{3}=4\times 1\times\sqrt{5}\times\sqrt{3}$
$=4\times\sqrt{5\times 3}$
$=4\sqrt{15}$

4. $7\sqrt{2}\times\sqrt{6}=7\times 1\times\sqrt{2}\times\sqrt{6}$
$=7\times\sqrt{2\times 6}$
$=7\sqrt{12}$
$=7\sqrt{4\times 3}$
$=7\sqrt{4}\times\sqrt{3}$
$=7\times 2\sqrt{3}$
$=14\sqrt{3}$

5. $2\sqrt{3}\times 4\sqrt{8}=2\times 4\times\sqrt{3}\times\sqrt{8}$
$=8\sqrt{3\times 8}$
$=8\sqrt{24}$
$=8\sqrt{4\times 6}$
$=8\sqrt{4}\times\sqrt{6}$
$=8\times 2\sqrt{6}$
$=16\sqrt{6}$

6. $5\sqrt{5}\times 3\sqrt{8}=5\times 3\times\sqrt{5}\times\sqrt{8}$
$=15\sqrt{5\times 8}$
$=15\sqrt{40}$
$=15\sqrt{4\times 10}$
$=15\sqrt{4}\times\sqrt{10}$
$=15\times 2\sqrt{10}$
$=30\sqrt{10}$

7. $\sqrt{3}\left(\sqrt{3}+\sqrt{8}\right)=\sqrt{3}\times\sqrt{3}+\sqrt{3}\times\sqrt{8}$
$=\sqrt{3\times 3}+\sqrt{3\times 8}$
$=3+\sqrt{24}$
$=3+\sqrt{4\times 6}$
$=3+\sqrt{4}\times\sqrt{6}$
$=3+2\sqrt{6}$

8. $\sqrt{11}\left(\sqrt{11}+\sqrt{8}\right)=\sqrt{11}\times\sqrt{11}+\sqrt{11}\times\sqrt{8}$
$$=\sqrt{11\times 11}+\sqrt{11\times 8}$$
$$=11+\sqrt{88}$$
$$=11+\sqrt{4\times 22}$$
$$=11+\sqrt{4}\times\sqrt{22}$$
$$=11+2\sqrt{22}$$

9. $10\left(8\sqrt{2}-\sqrt{6}\right)=10\times 8\sqrt{2}-10\times\sqrt{6}$
$$=80\sqrt{2}-10\sqrt{6}$$

10. $3\left(10\sqrt{5}-\sqrt{14}\right)=3\times 10\sqrt{5}-3\times\sqrt{14}$
$$=30\sqrt{5}-3\sqrt{14}$$

11. $3\sqrt{3}\left(5\sqrt{6}+2\sqrt{5}\right)=3\sqrt{3}\times 5\sqrt{6}+3\sqrt{3}\times 2\sqrt{5}$
$$=3\times 5\times\sqrt{3\times 6}+3\times 2\times\sqrt{3\times 5}$$
$$=15\sqrt{18}+6\sqrt{15}$$
$$=15\sqrt{9\times 2}+6\sqrt{15}$$
$$=15\sqrt{9}\times\sqrt{2}+6\sqrt{15}$$
$$=15\times 3\sqrt{2}+6\sqrt{15}$$
$$=45\sqrt{2}+6\sqrt{15}$$

12. $2\sqrt{5}\left(4\sqrt{7}+3\sqrt{6}\right)=2\sqrt{5}\times 4\sqrt{7}+2\sqrt{5}\times 3\sqrt{6}$
$$=2\times 4\times\sqrt{5\times 7}+2\times 3\times\sqrt{5\times 6}$$
$$=8\sqrt{35}+6\sqrt{30}$$

13. $\left(3\sqrt{5}-4\right)\left(3+2\sqrt{3}\right)=3\sqrt{5}\times 3+3\sqrt{5}\times 2\sqrt{3}-4\times 3-4\times 2\sqrt{3}$
$$=3\times 3\sqrt{5}+3\times 2\times\sqrt{5\times 3}-12-8\sqrt{3}$$
$$=9\sqrt{5}+6\sqrt{15}-12-8\sqrt{3}$$

14. $\left(5\sqrt{2}-6\right)\left(5+4\sqrt{2}\right)=5\sqrt{2}\times 5+5\sqrt{2}\times 4\sqrt{2}-6\times 5-6\times 4\sqrt{2}$
$$=5\times 5\sqrt{2}+5\times 4\times\sqrt{2\times 2}-30-24\sqrt{2}$$
$$=25\sqrt{2}+20\times 2-30-24\sqrt{2}$$
$$=40-30+25\sqrt{2}-24\sqrt{2}$$
$$=(40-30)+(25-24)\sqrt{2}$$
$$=10+\sqrt{2}$$

15. $\left(4\sqrt{3}+\sqrt{6}\right)^2=\left(4\sqrt{3}\right)^2+2\times 4\sqrt{3}\times\sqrt{6}+\left(\sqrt{6}\right)^2$
$$=16\times 3+2\times 4\times\sqrt{3\times 6}+\left(\sqrt{6}\right)^2$$
$$=16\times 3+2\times 4\sqrt{18}+6$$
$$=48+8\sqrt{18}+6$$
$$=54+8\sqrt{9\times 2}$$
$$=54+8\sqrt{9}\times\sqrt{2}$$
$$=54+24\sqrt{2}$$

16. $\left(3\sqrt{2}+\sqrt{5}\right)^2 = \left(3\sqrt{2}\right)^2 + 2\times 3\sqrt{2}\times\sqrt{5} + \left(\sqrt{5}\right)^2$

$= 9\times 2 + 2\times 3\sqrt{10} + 5$

$= 18 + 6\sqrt{10} + 5$

$= 23 + 6\sqrt{10}$

Activity 2

Multiply radicals which are conjugates of one another.

1. $\left(\sqrt{2}-\sqrt{3}\right)\left(\sqrt{2}+\sqrt{3}\right) = \left(\sqrt{2}\right)^2 + \sqrt{2}\times\sqrt{3} - \sqrt{3}\times\sqrt{2} - \left(\sqrt{3}\right)^2$

$= 2 + \sqrt{6} - \sqrt{6} - 3$

$= 2 - 3$

$= -1$

2. $\left(\sqrt{6}+\sqrt{5}\right)\left(\sqrt{6}-\sqrt{5}\right) = \left(\sqrt{6}\right)^2 - \left(\sqrt{5}\right)^2$

$= 6 - 5$

$= 1$

3. $\left(3\sqrt{3}-\sqrt{5}\right)\left(3\sqrt{3}+\sqrt{5}\right) = \left(3\sqrt{3}\right)^2 - \left(\sqrt{5}\right)^2$

$= 27 - 5$

$= 22$

4. $\left(4\sqrt{6}+\sqrt{2}\right)\left(4\sqrt{6}-\sqrt{2}\right) = \left(4\sqrt{6}\right)^2 - \left(\sqrt{2}\right)^2$

$= 96 - 2$

$= 94$

5. $\left(\sqrt{5}-2\sqrt{7}\right)\left(\sqrt{5}+2\sqrt{7}\right) = \left(\sqrt{5}\right)^2 - \left(2\sqrt{7}\right)^2$

$= 5 - 28$

$= -23$

6. $\left(\sqrt{6}+4\sqrt{2}\right)\left(\sqrt{6}-4\sqrt{2}\right) = \left(\sqrt{6}\right)^2 - \left(4\sqrt{2}\right)^2$

$= 6 - 32$

$= -26$

7. $\left(6\sqrt{3}-2\sqrt{10}\right)\left(6\sqrt{3}+2\sqrt{10}\right) = \left(6\sqrt{3}\right)^2 - \left(2\sqrt{10}\right)^2$

$= 108 - 40$

$= 68$

8. $\left(5\sqrt{13}+2\sqrt{11}\right)\left(5\sqrt{13}-2\sqrt{11}\right) = \left(5\sqrt{13}\right)^2 - \left(2\sqrt{11}\right)^2$

$= 325 - 44$

$= 281$

Activity 3

Divide radicals.

1. a. $$\frac{15\sqrt{45}}{3\sqrt{5}}=\frac{15}{3}\times\frac{\sqrt{45}}{\sqrt{5}}$$
$$=5\sqrt{\frac{45}{5}}$$
$$=5\sqrt{9}$$
$$=5\times3$$
$$=15$$

b. $$\frac{30\sqrt{50}}{6\sqrt{2}}=\frac{30}{6}\times\frac{\sqrt{50}}{\sqrt{2}}$$
$$=5\sqrt{\frac{50}{2}}$$
$$=5\sqrt{25}$$
$$=5\times5$$
$$=25$$

c. $$\frac{4\sqrt{6}}{\sqrt{3}}=\frac{4}{1}\times\frac{\sqrt{6}}{\sqrt{3}}$$
$$=4\sqrt{\frac{6}{3}}$$
$$=4\sqrt{2}$$

d. $$\frac{14\sqrt{22}}{\sqrt{8}}=\frac{14}{1}\times\sqrt{\frac{22}{8}}$$
$$=\frac{14}{1}\times\sqrt{\frac{11}{4}}$$
$$=\frac{14}{1}\times\frac{\sqrt{11}}{2}$$
$$=\frac{14\sqrt{11}}{2}$$
$$=7\sqrt{11}$$

2. a. $$\frac{2\sqrt{6}+10}{\sqrt{3}}=\frac{2\sqrt{6}+10}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$=\frac{2\sqrt{18}+10\sqrt{3}}{3}$$
$$=\frac{2\sqrt{9\times2}+10\sqrt{3}}{3}$$
$$=\frac{6\sqrt{2}+10\sqrt{3}}{3}$$

b. $$\frac{6\sqrt{3}-7}{\sqrt{5}}=\frac{6\sqrt{3}-7}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$$
$$=\frac{6\sqrt{15}-7\sqrt{5}}{5}$$

c. $$\frac{\sqrt{5}-\sqrt{2}}{3\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$
$$=\frac{\sqrt{10}-2}{3\times 2}$$
$$=\frac{\sqrt{10}-2}{6}$$

d. $$\frac{\sqrt{10}+\sqrt{6}}{5\sqrt{3}}=\frac{\sqrt{10}+\sqrt{6}}{5\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$=\frac{\sqrt{30}+\sqrt{18}}{5\times 3}$$
$$=\frac{\sqrt{30}+3\sqrt{2}}{15}$$

3. $A = lw$
$$=\left(\frac{\sqrt{10}+\sqrt{2}}{\sqrt{5}}\right)\times\left(\sqrt{10}-\sqrt{2}\right)$$
$$=\frac{\left(\sqrt{10}+\sqrt{2}\right)\left(\sqrt{10}-\sqrt{2}\right)}{\sqrt{5}}$$
$$=\frac{10-2}{\sqrt{5}}$$
$$=\frac{8}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$$
$$=\frac{8\sqrt{5}}{5}$$

The expression for the area is $\frac{8\sqrt{5}}{5}$ square units.

Extra Help

1. a. $$\sqrt{6}\times\sqrt{2}=\sqrt{12}$$
$$=\sqrt{4\times 3}$$
$$=2\sqrt{3}$$

b. $$\sqrt{5}\times\sqrt{6}\times\sqrt{2}=\sqrt{60}$$
$$=\sqrt{4\times 15}$$
$$=2\sqrt{15}$$

c. $$4\sqrt{3}\times 2\sqrt{6} = 4\times 2\times\sqrt{3}\times\sqrt{6}$$
$$= 8\sqrt{18}$$
$$= 8\sqrt{9\times 2}$$
$$= 8\times 3\sqrt{2}$$
$$= 24\sqrt{2}$$

d. $$2\sqrt{2}\times 3\sqrt{5}\times 6\sqrt{2} = 2\times 3\times 6\times\sqrt{2}\times\sqrt{5}\times\sqrt{2}$$
$$= 36\sqrt{20}$$
$$= 36\sqrt{4\times 5}$$
$$= 36\times 2\sqrt{5}$$
$$= 72\sqrt{5}$$

e. $$5\sqrt{10}\times 4\sqrt{5}\times 3\sqrt{2} = 5\times 4\times 3\times\sqrt{10}\times\sqrt{5}\times\sqrt{2}$$
$$= 60\sqrt{100}$$
$$= 60\times 10$$
$$= 600$$

2. a. $$\frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}}$$
$$= \sqrt{5}$$

b. $$\frac{\sqrt{24}}{\sqrt{3}} = \sqrt{\frac{24}{3}}$$
$$= \sqrt{8}$$
$$= \sqrt{4\times 2}$$
$$= 2\sqrt{2}$$

c. $$\frac{3\sqrt{50}}{\sqrt{2}} = 3\sqrt{\frac{50}{2}}$$
$$= 3\sqrt{25}$$
$$= 3\times 5$$
$$= 15$$

d. $$\frac{16\sqrt{72}}{4\sqrt{8}} = \frac{16}{4}\times\frac{\sqrt{72}}{\sqrt{8}}$$
$$= 4\sqrt{\frac{72}{8}}$$
$$= 4\times\sqrt{9}$$
$$= 4\times 3$$
$$= 12$$

e. $$\frac{6\sqrt{10}+2\sqrt{6}}{2\sqrt{2}}=\frac{6\sqrt{10}}{2\sqrt{2}}+\frac{2\sqrt{6}}{2\sqrt{2}}$$
$$=\frac{6}{2}\times\frac{\sqrt{10}}{\sqrt{2}}+\frac{2}{2}\times\frac{\sqrt{6}}{\sqrt{2}}$$
$$=3\sqrt{5}+1\times\sqrt{3}$$
$$=3\sqrt{5}+\sqrt{3}$$

3. a. $$\frac{3}{\sqrt{2}}=\frac{3}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$
$$=\frac{3\sqrt{2}}{2}$$

b. $$\frac{5}{\sqrt{5}}=\frac{5}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$$
$$=\frac{5\sqrt{5}}{5}$$
$$=\sqrt{5}$$

c. $$\frac{\sqrt{3}}{\sqrt{5}}=\frac{\sqrt{3}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$$
$$=\frac{\sqrt{15}}{5}$$

d. $$\frac{2\sqrt{5}}{\sqrt{3}}=\frac{2\sqrt{5}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$=\frac{2\sqrt{15}}{3}$$

e. $$\frac{2\sqrt{6}}{\sqrt{3}}=\frac{2\sqrt{6}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$=\frac{2\sqrt{18}}{3}$$
$$=\frac{2\sqrt{9\times2}}{3}$$
$$=\frac{2\times3\sqrt{2}}{3}$$
$$=2\sqrt{2}$$

Extensions

1. a. $$2\sqrt{a}\left(2\sqrt{a}-5\sqrt{ay}\right)=\left(2\sqrt{a}\right)\left(2\sqrt{a}\right)-\left(2\sqrt{a}\right)\left(5\sqrt{ay}\right)$$
$$=4\sqrt{a^2}-10\sqrt{a^2y}$$
$$=4a-10a\sqrt{y}$$

b. $\left(5\sqrt{x}-\sqrt{3y}\right)\left(\sqrt{4x}+\sqrt{5y}\right)=\left(5\sqrt{x}\right)\left(\sqrt{4x}\right)+\left(5\sqrt{x}\right)\left(\sqrt{5y}\right)-\left(\sqrt{3y}\right)\left(\sqrt{4x}\right)-\left(\sqrt{3y}\right)\left(\sqrt{5y}\right)$

$$=5\sqrt{4x^2}+5\sqrt{5xy}-\sqrt{12xy}-\sqrt{15y^2}$$
$$=5(2x)+5\sqrt{5xy}-2\sqrt{3xy}-y\sqrt{15}$$
$$=10x+5\sqrt{5xy}-2\sqrt{3xy}-y\sqrt{15}$$

c. $\left(4\sqrt{x}-\sqrt{y}+\sqrt{z}\right)\left(4\sqrt{x}+\sqrt{y}-\sqrt{z}\right)=\left(4\sqrt{x}\right)\left(4\sqrt{x}\right)+\left(4\sqrt{x}\right)\left(\sqrt{y}\right)-\left(4\sqrt{x}\right)\left(\sqrt{z}\right)-\left(\sqrt{y}\right)\left(4\sqrt{x}\right)-\left(\sqrt{y}\right)\left(\sqrt{y}\right)$

$$+\left(\sqrt{y}\right)\left(\sqrt{z}\right)+\left(\sqrt{z}\right)\left(4\sqrt{x}\right)+\left(\sqrt{z}\right)\left(\sqrt{y}\right)-\left(\sqrt{z}\right)\left(\sqrt{z}\right)$$
$$=16\sqrt{x^2}+4\sqrt{xy}-4\sqrt{xz}-4\sqrt{xy}-\sqrt{y^2}+\sqrt{yz}+4\sqrt{xz}+\sqrt{yz}-\sqrt{z^2}$$
$$=16x+4\sqrt{xy}-4\sqrt{xz}-4\sqrt{xy}-y+\sqrt{yz}+4\sqrt{xz}+\sqrt{yz}-z$$
$$=16x+4\sqrt{xy}-4\sqrt{xy}-4\sqrt{xz}+4\sqrt{xz}-y+\sqrt{yz}+\sqrt{yz}-z$$
$$=16x-y+2\sqrt{yz}-z$$

d. $\left(\sqrt{x}-\sqrt{y}\right)\left(\sqrt{x}+\sqrt{y}\right)(x+y)\left(x^2+y^2\right)=\left[\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2\right](x+y)\left(x^2+y^2\right)$

$$=(x-y)(x+y)\left(x^2+y^2\right)$$
$$=\left(x^2-y^2\right)\left(x^2+y^2\right)$$
$$=x^4-y^4$$

e. $$\left(5\sqrt{x}-2\sqrt{y}\right)\left(3\sqrt{x}+5\sqrt{y}\right)=\left(5\sqrt{x}\right)\left(3\sqrt{x}\right)+\left(5\sqrt{x}\right)\left(5\sqrt{y}\right)-\left(2\sqrt{y}\right)\left(3\sqrt{x}\right)-\left(2\sqrt{y}\right)\left(5\sqrt{y}\right)$$
$$=15\sqrt{x^2}+25\sqrt{xy}-6\sqrt{xy}-10\sqrt{y^2}$$
$$=15x+19\sqrt{xy}-10y$$

f. $$\left(3\sqrt{5x}+2\sqrt{3y}\right)\left(3\sqrt{5x}-2\sqrt{3y}\right)=\left(3\sqrt{5x}\right)\left(3\sqrt{5x}\right)-\left(3\sqrt{5x}\right)\left(2\sqrt{3y}\right)+\left(2\sqrt{3y}\right)\left(3\sqrt{5x}\right)-\left(2\sqrt{3y}\right)\left(2\sqrt{3y}\right)$$
$$=9\sqrt{25x^2}-6\sqrt{15xy}+6\sqrt{15xy}-4\sqrt{9y^2}$$
$$=9(5x)-4(3y)$$
$$=45x-12y$$

2. a. $$\frac{16x^2\sqrt{7}}{4x}=\frac{16}{4}\times\frac{x^2}{x}\times\frac{\sqrt{7}}{1}$$
$$=4x\sqrt{7}$$

b. $$\frac{50a^3\sqrt{21}}{5a\sqrt{7}}=\frac{50}{5}\times\frac{a^3}{a}\times\frac{\sqrt{21}}{\sqrt{7}}$$
$$=10a^2\sqrt{3}$$

c. $$\frac{72b^4\sqrt{27b^2}}{9b^5}=\frac{72b^4\sqrt{9\times3\times b^2}}{9b^5}$$
$$=\frac{216b^5\sqrt{3}}{9b^5}$$
$$=\frac{216}{9}\times\frac{b^5}{b^5}\times\frac{\sqrt{3}}{1}$$
$$=24\sqrt{3}$$

d. $$\frac{100x\sqrt{50y^3}}{\sqrt{50y^2}}=\frac{100x\sqrt{25\times 2\times y^2\times y}}{\sqrt{25\times 2\times y^2}}$$
$$=\frac{500xy\sqrt{2y}}{5y\sqrt{2}}$$
$$=\frac{500}{5}\times\frac{xy}{y}\times\frac{\sqrt{2y}}{\sqrt{2}}$$
$$=100x\sqrt{y}$$

or

$$\frac{100x\sqrt{50y^3}}{\sqrt{50y^2}}=100x\sqrt{\frac{50y^3}{50y^2}}$$
$$=100x\sqrt{y}$$

e. $$\frac{2abc\sqrt{72}}{16ac}=\frac{2abc\sqrt{36\times 2}}{16ac}$$
$$=\frac{12abc\sqrt{2}}{16ac}$$
$$=\frac{12}{16}\times\frac{abc}{ac}\times\frac{\sqrt{2}}{1}$$
$$=\frac{3}{4}b\sqrt{2}$$

f. $$\frac{\sqrt{1250x^3y^4}}{10xy^2}=\frac{\sqrt{625\times 2\times x^2\times x\times y^2\times y^2}}{10xy^2}$$
$$=\frac{25xy^2\sqrt{2x}}{10xy^2}$$
$$=\frac{25}{10}\times\frac{xy^2}{xy^2}\times\frac{\sqrt{2x}}{1}$$
$$=\frac{5}{2}\sqrt{2x}$$

3. a. $$\frac{3\sqrt{a}}{\sqrt{3}}=\frac{3\sqrt{a}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$=\frac{3\sqrt{3a}}{3}$$
$$=\frac{3}{3}\times\frac{\sqrt{3a}}{1}$$
$$=\sqrt{3a}$$

b. $$\frac{5}{\sqrt{x}+1}=\frac{5}{\sqrt{x}+1}\times\frac{\sqrt{x}-1}{\sqrt{x}-1}$$
$$=\frac{5\sqrt{x}-5}{x-1}$$

c. $$\frac{\sqrt{5}}{\sqrt{3}+\sqrt{x}}=\frac{\sqrt{5}}{\sqrt{3}+\sqrt{x}}\times\frac{\sqrt{3}-\sqrt{x}}{\sqrt{3}-\sqrt{x}}$$
$$=\frac{\sqrt{15}-\sqrt{5x}}{3-x}$$

d. $$\frac{24}{3\sqrt{a}-5\sqrt{2}}=\frac{24}{3\sqrt{a}-5\sqrt{2}}\times\frac{3\sqrt{a}+5\sqrt{2}}{3\sqrt{a}+5\sqrt{2}}$$
$$=\frac{72\sqrt{a}+120\sqrt{2}}{9a-50}$$

e. $$\frac{4+3\sqrt{a}}{-6-2\sqrt{a}}=\frac{4+3\sqrt{a}}{-6-2\sqrt{a}}\times\frac{-6+2\sqrt{a}}{-6+2\sqrt{a}}$$
$$=\frac{-24+8\sqrt{a}-18\sqrt{a}+6\sqrt{a^2}}{36-4\sqrt{a^2}}$$
$$=\frac{-24-10\sqrt{a}+6a}{36-4a}$$
$$=\frac{2\left(-12-5\sqrt{a}+3a\right)}{2(18-2a)}$$
$$=\frac{-12-5\sqrt{a}+3a}{18-2a}$$

f. $$\frac{3\sqrt{y}-2\sqrt{x}}{4\sqrt{y}-3\sqrt{x}}=\frac{3\sqrt{y}-2\sqrt{x}}{4\sqrt{y}-3\sqrt{x}}\times\frac{4\sqrt{y}+3\sqrt{x}}{4\sqrt{y}+3\sqrt{x}}$$
$$=\frac{12\sqrt{y^2}+9\sqrt{xy}-8\sqrt{xy}-6\sqrt{x^2}}{16\sqrt{y^2}-9\sqrt{x^2}}$$
$$=\frac{12y+\sqrt{xy}-6x}{16y-9x}$$

Exploring Topic 3

Activity 1

Solve and verify radical equations involving a single radical, and identify possible solutions of radical equations as being extraneous.

1. $$\sqrt{x}=4$$
$$\left(\sqrt{x}\right)^2=(4)^2$$
$$x=16$$

Check: $x=16$

LS	RS
$\sqrt{x}$	4
$\sqrt{16}$	4
4	4

LS = RS
(checks)

The solution is 16.

2. $$3\sqrt{k}=15$$
$$\left(3\sqrt{k}\right)^2=(15)^2$$
$$9k=225$$
$$\frac{9k}{9}=\frac{225}{9}$$
$$k=25$$

Check: $k=25$

LS	RS
$3\sqrt{k}$	15
$3\sqrt{25}$	15
3×5	15
15	15

LS = RS
(checks)

The solution is 25.

3. $\sqrt{y+5}=-1$

$$\left(\sqrt{y+5}\right)^2=(-1)^2$$
$$y+5=1$$
$$y+5-5=1-5$$
$$y=-4$$

Check:

$y=-4$

LS	RS
$\sqrt{y+5}$	-1
$\sqrt{-4+5}$	-1
$\sqrt{1}$	-1
1	-1

LS $\neq$ RS
(does not check)

The solution is extraneous. There is no root.

4. $\sqrt{x-4}=6$

$$\left(\sqrt{x-4}\right)^2=(6)^2$$
$$x-4=36$$
$$x-4+4=36+4$$
$$x=40$$

Check:

$x=40$

LS	RS
$\sqrt{x-4}$	6
$\sqrt{40-4}$	6
$\sqrt{36}$	6
6	6

LS = RS
(checks)

The solution is 40.

5. $\sqrt{x+6}-3=1$

$$\sqrt{x+6}-3+3=1+3$$
$$\sqrt{x+6}=4$$
$$\left(\sqrt{x+6}\right)^2=4^2$$
$$x+6=16$$
$$x+6-6=16-6$$
$$x=10$$

Check:

$x=10$

LS	RS
$\sqrt{x+6}-3$	1
$\sqrt{10+6}-3$	1
$\sqrt{16}-3$	1
$4-3$	1
1	1

LS = RS
(checks)

The solution is 10.

6. $\sqrt{2h+1}+4=3$

$$\sqrt{2h+1}+4-4=3-4$$
$$\sqrt{2h+1}=-1$$
$$\left(\sqrt{2h+1}\right)^2=(-1)^2$$
$$2h+1=1$$
$$2h+1-1=1-1$$
$$2h=0$$
$$h=0$$

Check:

$h=0$

LS	RS
$\sqrt{2h+1}+4$	3
$\sqrt{2(0)+1}+4$	3
$\sqrt{0+1}+4$	3
$\sqrt{1}+4$	3
$1+4$	3
5	3

LS $\neq$ RS
(does not check)

The solution is extraneous. There is no root.

7.
$$\begin{aligned}\sqrt{x-1}-x+7&=0\\ \sqrt{x-1}-x+x+7-7&=x-7\\ \sqrt{x-1}&=x-7\\ \left(\sqrt{x-1}\right)^2&=(x-7)^2\\ x-1&=x^2-14x+49\\ x^2-14x+49-x+1&=0\\ x^2-15x+50&=0\\ (x-5)(x-10)&=0\end{aligned}$$

$x-5=0$ or $x-10=0$

$x=5$ $\quad$ $x=10$

Check:

$x=5$

LS	RS
$\sqrt{x-1}-x+7$	0
$\sqrt{5-1}-5+7$	0
$\sqrt{4}-5+7$	0
$2-5+7$	0
$9-5$	0
4	0

LS $\neq$ RS
(does not check)

Check:

$x=10$

LS	RS
$\sqrt{x-1}-x+7$	0
$\sqrt{10-1}-10+7$	0
$\sqrt{9}-10+7$	0
$3-10+7$	0
$10-10$	0
0	0

LS = RS
(checks)

The solution is 10 while 5 is an extraneous root.

8.
$$\begin{aligned}2\sqrt{x}&=x-3\\ \left(2\sqrt{x}\right)^2&=(x-3)^2\\ 4x&=x^2-6x+9\\ x^2-6x+9-4x&=4x-4x\\ x^2-10x+9&=0\\ (x-9)(x-1)&=0\end{aligned}$$

$x-9=0$ or $x-1=0$

$x=9$ $\quad$ $x=1$

Check:

$x = 9$

LS	RS
$2\sqrt{x}$	$x - 3$
$2\sqrt{9}$	$9 - 3$
2×3	6
6	6

LS = RS
(checks)

$x = 1$

LS	RS
$2\sqrt{x}$	$x - 3$
$2\sqrt{1}$	$1 - 3$
2×1	-2
2	-2

LS $\neq$ RS
(does not check)

The solution is 9 while 1 is an extraneous root.

Activity 2

Apply simple radical equations to solve problems.

1. Let x be the required number.

$$\sqrt{x+1} = 3$$
$$\left(\sqrt{x+1}\right)^2 = 3^2$$
$$x+1 = 9$$
$$x+1-1 = 9-1$$
$$x = 8$$

Check:

$x = 8$

LS	RS
$\sqrt{x+1}$	3
$\sqrt{8+1}$	3
$\sqrt{9}$	3
3	3

LS = RS
(checks)

The required number is 8.

2. Let x be the number of cars in the showroom.

$$\sqrt{x^2+9} = 2x-3$$
$$\left(\sqrt{x^2+9}\right)^2 = (2x-3)^2$$
$$x^2+9 = 4x^2-12x+9$$
$$4x^2-x^2-12x+9-9 = 0$$
$$3x^2-12x = 0$$
$$3x(x-4) = 0$$
$$3x = 0 \quad \text{or} \quad x-4 = 0$$
$$x = 0 \qquad\qquad x = 4$$

Check:

$x = 4$

LS	RS
$\sqrt{x^2+9}$	$2x-3$
$\sqrt{(4)^2+9}$	$2(4)-3$
$\sqrt{16+9}$	$8-3$
$\sqrt{25}$	5
5	5

LS = RS
(checks)

$x = 0$

LS	RS
$\sqrt{x^2+9}$	$2x-3$
$\sqrt{(0)^2+9}$	$2(0)-3$
$\sqrt{9}$	$0-3$
3	-3

LS ≠ RS
(does not check)

The $x = 0$ root is extraneous since there must be some cars in the showroom.

The number of cars in the showroom is 4.

3. Set up the equation and solve for y.

$$\sqrt{2y+1}+\frac{y}{2}=5$$

$$2\left(\sqrt{2y+1}\right)+2\left(\frac{y}{2}\right)=2(5) \qquad \text{(Multiply each term by 2.)}$$

$$2\sqrt{2y+1}+y=10$$

$$2\sqrt{2y+1}=10-y$$

$$\left(2\sqrt{2y+1}\right)^2=(10-y)^2$$

$$4(2y+1)=100-20y+y^2$$

$$8y+4=y^2-20y+100$$

$$y^2-20y-8y+100-4=0$$

$$y^2-28y+96=0$$

$$(y-24)(y-4)=0$$

$$y-24=0 \quad \text{or} \quad y-4=0$$

$$y=24 \qquad\qquad y=4$$

The root $y = 24$ does not need to be verified since there is no positive number which when added to 24 will result in 5.
The root $y = 24$ is extraneous since 24 is greater than 5.

Check:

$y = 4$

LS	RS
$\sqrt{2y+1}+\frac{y}{2}$	5
$\sqrt{2(4)+1}+\frac{4}{2}$	5
$\sqrt{8+1}+2$	5
$\sqrt{9}+2$	5
$3+2$	5
5	5

LS = RS
(checks)

The value for y is 4.

For one statue, $\sqrt{2y+1}=\sqrt{2(4)+1}$

$$=\sqrt{8+1}$$
$$=\sqrt{9}$$
$$=3$$

For the other statue, $\frac{y}{2}=\frac{4}{2}$

$$=2$$

The two statues are 3 m and 2 m in height.

4. Let the number be x.

$$3\sqrt{x}-7=8$$
$$3\sqrt{x}-7+7=8+7$$
$$3\sqrt{x}=15$$
$$\frac{3\sqrt{x}}{3}=\frac{15}{3}$$
$$\sqrt{x}=5$$
$$\left(\sqrt{x}\right)^2=(5)^2$$
$$x=25$$

Check:

$x = 25$

LS	RS
$3\sqrt{x}-7$	8
$3\sqrt{25}-7$	8
$3(5)-7$	8
$15-7$	8
8	8

LS = RS
(checks)

The required number is 25.

5. Let n be the number.

$$
\begin{aligned}
3\sqrt{2n+1} &= 15 \\
\frac{3\sqrt{2n+1}}{3} &= \frac{15}{3} \\
\sqrt{2n+1} &= 5 \\
\left(\sqrt{2n+1}\right)^2 &= (5)^2 \\
2n+1 &= 25 \\
2n &= 24 \\
n &= 12
\end{aligned}
$$

Check:

$n = 12$

LS	RS
$3\sqrt{2n+1}$	15
$3\sqrt{2(12)+1}$	15
$3\sqrt{24+1}$	15
$3\sqrt{25}$	15
$3(5)$	15
15	15

LS = RS
(checks)

The required answer to the skill-testing question is 12.

Extra Help

1. a.

$$
\begin{aligned}
\sqrt{x} &= 6 \\
\left(\sqrt{x}\right)^2 &= (6)^2 \\
x &= 36
\end{aligned}
$$

Check:

$x = 36$

LS	RS
$\sqrt{x}$	6
$\sqrt{36}$	6
6	6

LS = RS
(checks)

The solution is 36.

b.

$$
\begin{aligned}
\sqrt{m} &= 3 \\
\left(\sqrt{m}\right)^2 &= (3)^2 \\
m &= 9
\end{aligned}
$$

Check:

$m = 9$

LS	RS
$\sqrt{m}$	3
$\sqrt{9}$	3
3	3

LS = RS
(checks)

The solution is 9.

c.

$$
\begin{aligned}
3\sqrt{x} - 2 &= 4 \\
3\sqrt{x} - 2 + 2 &= 4 + 2 \\
3\sqrt{x} &= 6 \\
\left(3\sqrt{x}\right)^2 &= (6)^2 \\
9x &= 36 \\
\frac{9x}{9} &= \frac{36}{9} \\
x &= 4
\end{aligned}
$$

Check:

$x = 4$

LS	RS
$3\sqrt{x} - 2$	4
$3\sqrt{4} - 2$	4
$3(2) - 2$	4
$6 - 2$	4
4	4

LS = RS
(checks)

The solution is 4.

d.

$$3\sqrt{2x+1}=15$$
$$\left(3\sqrt{2x+1}\right)^2=(15)^2$$
$$9(2x+1)=225$$
$$18x+9=225$$
$$18x+9-9=225-9$$
$$18x=216$$
$$\frac{18x}{18}=\frac{216}{18}$$
$$x=12$$

Check:

$x = 12$

LS	RS
$3\sqrt{2x+1}$	15
$3\sqrt{2(12)+1}$	15
$3\sqrt{24+1}$	15
$3\sqrt{25}$	15
$3(5)$	15
15	15

LS = RS
(checks)

The solution is 12.

e.

$$\sqrt{5x-1}-1=x$$
$$\sqrt{5x-1}-1+1=x+1$$
$$\sqrt{5x-1}=x+1$$
$$\left(\sqrt{5x-1}\right)^2=(x+1)^2$$
$$5x-1=x^2+2x+1$$
$$x^2-3x+2=0$$
$$(x-2)(x-1)=0$$
$$x-2=0 \quad \text{or} \quad x-1=0$$
$$x=2 \qquad\qquad x=1$$

Check:

$x = 2$

LS	RS
$\sqrt{5x-1}-1$	x
$\sqrt{5(2)-1}-1$	2
$\sqrt{10-1}-1$	2
$\sqrt{9}-1$	2
$3-1$	2
2	2

LS = RS
(checks)

Check:

$x = 1$

LS	RS
$\sqrt{5x-1}-1$	x
$\sqrt{5(1)-1}-1$	1
$\sqrt{5-1}-1$	1
$\sqrt{4}-1$	1
$2-1$	1
1	1

LS = RS
(checks)

The solutions are 1 and 2.

f.

$$\sqrt{2n^2-n+4}-n=2$$
$$\sqrt{2n^2-n+4}-n+n=2+n$$
$$\sqrt{2n^2-n+4}=n+2$$
$$\left(\sqrt{2n^2-n+4}\right)^2=(n+2)^2$$
$$2n^2-n+4=n^2+4n+4$$
$$2n^2-n^2-n-4n+4-4=n^2-n^2+4n-4n+4-4$$
$$n^2-5n=0$$
$$n(n-5)=0$$
$$n=0 \quad \text{or} \quad n-5=0$$
$$n=5$$

Check:

$n = 0$

LS	RS
$\sqrt{2n^2 - n + 4} - n$	2
$\sqrt{2(0)^2 - 0 + 4} - 0$	2
$\sqrt{2(0) + 4}$	2
$\sqrt{4}$	2
2	2

LS = RS
(checks)

$n = 5$

LS	RS
$\sqrt{2n^2 - n + 4} - n$	2
$\sqrt{2(5)^2 - 5 + 4} - 5$	2
$\sqrt{50 - 5 + 4} - 5$	2
$\sqrt{49} - 5$	2
$7 - 5$	2
2	2

LS = RS
(checks)

The solutions are 0 and 5.

g.

$$\sqrt{5p+4} = 5 - 2p$$

$$\left(\sqrt{5p+4}\right)^2 = (5-2p)^2$$

$$5p + 4 = 25 - 20p + 4p^2$$

$$5p - 5p + 4 - 4 = 25 - 4 - 20p - 5p + 4p^2$$

$$4p^2 - 25p + 21 = 0$$

$$(4p - 21)(p - 1) = 0$$

$$4p - 21 = 0 \quad \text{or} \quad p - 1 = 0$$

$$4p = 21 \qquad p = 1$$

$$p = \frac{21}{4}$$

Check:

$p = \frac{21}{4}$

LS	RS
$\sqrt{5p+4}$	$5 - 2p$
$\sqrt{5\left(\frac{21}{4}\right) + 4}$	$5 - 2\left(\frac{21}{4}\right)$
$\sqrt{\frac{105}{4} + 4}$	$5 - \frac{42}{4}$
$\sqrt{\frac{105}{4} + \frac{16}{4}}$	$\frac{20}{4} - \frac{42}{4}$
$\sqrt{\frac{121}{4}}$	$\frac{-22}{4}$
$\frac{11}{2}$	$\frac{-11}{2}$

LS $\neq$ RS
(does not check)

$p = 1$

LS	RS
$\sqrt{5p+4}$	$5 - 2p$
$\sqrt{5(1) + 4}$	$5 - 2(1)$
$\sqrt{5+4}$	$5 - 2$
$\sqrt{9}$	3
3	3

LS = RS
(checks)

The solution is 1. The value $\frac{21}{4}$ does not check; therefore, it is an extraneous root.

h.
$$\begin{aligned}
4+2\sqrt{5x-3} &= 12\\
4-4+2\sqrt{5x-3} &= 12-4\\
2\sqrt{5x-3} &= 8\\
\left(2\sqrt{5x-3}\right)^2 &= (8)^2\\
4(5x-3) &= 64\\
20x-12 &= 64\\
20x &= 76\\
x &= \frac{76}{20}\\
x &= \frac{19}{5}
\end{aligned}$$

Check:

$x=\frac{19}{5}$

LS	RS
$4+2\sqrt{5x-3}$	12
$4+2\sqrt{5\left(\frac{19}{5}\right)-3}$	12
$4+2\sqrt{19-3}$	12
$4+2\sqrt{16}$	12
$4+2(4)$	12
$4+8$	12
12	12

LS = RS
(checks)

The solution is $\frac{19}{5}$.

2. Let x be the number.
$$\begin{aligned}
3+\sqrt{2x-3} &= x\\
-3+3+\sqrt{2x-3} &= x-3\\
\sqrt{2x-3} &= x-3\\
\left(\sqrt{2x-3}\right)^2 &= (x-3)^2\\
2x-3 &= x^2-6x+9\\
2x-2x-3+3 &= x^2-6x-2x+9+3\\
x^2-8x+12 &= 0\\
(x-6)(x-2) &= 0
\end{aligned}$$

$x-6=0$ or $x-2=0$

$x=6$ $\quad$ $x=2$

Check:

$x=6$

LS	RS
$3+\sqrt{2x-3}$	x
$3+\sqrt{2(6)-3}$	6
$3+\sqrt{12-3}$	6
$3+\sqrt{9}$	6
$3+3$	6
6	6

LS = RS
(checks)

$x=2$

LS	RS
$3+\sqrt{2x-3}$	x
$3+\sqrt{2(2)-3}$	2
$3+\sqrt{4-3}$	2
$3+\sqrt{1}$	2
$3+1$	2
4	2

LS $\neq$ RS
(does not check)

The required number is 6. The other root, $x=2$, is an extraneous root since the conditions of the problem are not satisfied.

Extensions

1. a. $$\begin{aligned}\sqrt{2y+5}-\sqrt{y-2}&=3\\ \sqrt{2y+5}&=3+\sqrt{y-2}\\ \left(\sqrt{2y+5}\right)^2&=\left(3+\sqrt{y-2}\right)^2\\ 2y+5&=9+6\sqrt{y-2}+y-2\\ 2y+5&=7+y+6\sqrt{y-2}\\ y-2&=6\sqrt{y-2}\\ (y-2)^2&=\left(6\sqrt{y-2}\right)^2\\ y^2-4y+4&=36(y-2)\\ y^2-4y+4&=36y-72\\ y^2-40y+76&=0\\ (y-38)(y-2)&=0\end{aligned}$$

$y-38=0$ or $y-2=0$

$y=38 \qquad y=2$

Check:

$y=38$

LS	RS
$\sqrt{2y+5}-\sqrt{y-2}$	3
$\sqrt{2(38)+5}-\sqrt{38-2}$	3
$\sqrt{76+5}-\sqrt{36}$	3
$\sqrt{81}-6$	3
$9-6$	3
3	3

LS = RS
(checks)

$y=2$

LS	RS
$\sqrt{2y+5}-\sqrt{y-2}$	3
$\sqrt{2(2)+5}-\sqrt{2-2}$	3
$\sqrt{4+5}-\sqrt{0}$	3
$\sqrt{9}-0$	3
$3-0$	3
3	3

LS = RS
(checks)

The solutions are 38 and 2.

b.

$$\sqrt{4-6x}-1=\sqrt{-5x-1}$$
$$\sqrt{4-6x}=1+\sqrt{-5x-1}$$
$$\left(\sqrt{4-6x}\right)^2=\left(1+\sqrt{-5x-1}\right)^2$$
$$4-6x=1+2\sqrt{-5x-1}-5x-1$$
$$4-6x=-5x+2\sqrt{-5x-1}$$
$$4-x=2\sqrt{-5x-1}$$
$$(4-x)^2=\left(2\sqrt{-5x-1}\right)^2$$
$$16-8x+x^2=4(-5x-1)$$
$$x^2-8x+16=-20x-4$$
$$x^2+12x+20=0$$
$$(x+2)(x+10)=0$$
$$x+2=0 \quad \text{or} \quad x+10=0$$
$$x=-2 \qquad\qquad x=-10$$

Check:

$x=-2$

LS	RS
$\sqrt{4-6x}-1$	$\sqrt{-5x-1}$
$\sqrt{4-6(-2)}-1$	$\sqrt{-5(-2)-1}$
$\sqrt{4+12}-1$	$\sqrt{10-1}$
$\sqrt{16}-1$	$\sqrt{9}$
$4-1$	3
3	3

LS = RS
(checks)

$x=-10$

LS	RS
$\sqrt{4-6x}-1$	$\sqrt{-5x-1}$
$\sqrt{4-6(-10)}-1$	$\sqrt{-5(-10)-1}$
$\sqrt{4+60}-1$	$\sqrt{50-1}$
$\sqrt{64}-1$	$\sqrt{49}$
$8-1$	7
7	7

LS = RS
(checks)

The solutions are -2 and -10.

c. $\sqrt{3-x}+\sqrt{2x+3}=3$

$$\begin{aligned}
\sqrt{3-x} &= 3-\sqrt{2x+3} \\
\left(\sqrt{3-x}\right)^2 &= \left(3-\sqrt{2x+3}\right)^2 \\
3-x &= 9-6\sqrt{2x+3}+2x+3 \\
3-x &= 12+2x-6\sqrt{2x+3} \\
-3x-9 &= -6\sqrt{2x+3} \\
(-3x-9)^2 &= \left(-6\sqrt{2x+3}\right)^2 \\
9x^2+54x+81 &= 36(2x+3) \\
9x^2+54x+81 &= 72x+108 \\
9x^2-18x-27 &= 0 \\
9\left(x^2-2x-3\right) &= 0 \\
x^2-2x-3 &= 0 \\
(x-3)(x+1) &= 0
\end{aligned}$$

$x+1=0$ or $x-3=0$

$x=-1$ $\quad x=3$

Check:

$x=-1$

LS	RS
$\sqrt{3-x}+\sqrt{2x+3}$	3
$\sqrt{3-(-1)}+\sqrt{2(-1)+3}$	3
$\sqrt{3+1}+\sqrt{-2+3}$	3
$\sqrt{4}+\sqrt{1}$	3
$2+1$	3
3	3

LS = RS
(checks)

$x=3$

LS	RS
$\sqrt{3-x}+\sqrt{2x+3}$	3
$\sqrt{3-3}+\sqrt{2(3)+3}$	3
$\sqrt{0}+\sqrt{6+3}$	3
$0+\sqrt{9}$	3
$0+3$	3
3	3

LS = RS
(checks)

The solutions are 3 and -1.

d. $\sqrt{x+9}-\sqrt{x-6}=3$

$$
\begin{aligned}
\sqrt{x+9} &= 3+\sqrt{x-6} \\
\left(\sqrt{x+9}\right)^2 &= \left(3+\sqrt{x-6}\right)^2 \\
x+9 &= 9+6\sqrt{x-6}+x-6 \\
x+9 &= 3+x+6\sqrt{x-6} \\
6 &= 6\sqrt{x-6} \\
(6)^2 &= \left(6\sqrt{x-6}\right)^2 \\
36 &= 36(x-6) \\
36 &= 36x-216 \\
36x-216 &= 36 \\
36x-252 &= 0 \\
36(x-7) &= 0 \\
x-7 &= 0 \\
x &= 7
\end{aligned}
$$

Check:

$x=7$

LS	RS
$\sqrt{x+9}-\sqrt{x-6}$	3
$\sqrt{7+9}-\sqrt{7-6}$	3
$\sqrt{16}-\sqrt{1}$	3
$4-1$	3
3	3

LS = RS
(checks)

The solution is 7.

e. $\dfrac{1}{\sqrt{x+1}}=\dfrac{\sqrt{2x+3}}{2x}$

$$
\begin{aligned}
2x &= \left(\sqrt{x+1}\right)\left(\sqrt{2x+3}\right) \\
2x &= \sqrt{2x^2+5x+3} \\
(2x)^2 &= \left(\sqrt{2x^2+5x+3}\right)^2 \\
4x^2 &= 2x^2+5x+3 \\
2x^2-5x-3 &= 0 \\
(2x+1)(x-3) &= 0
\end{aligned}
$$

$2x+1=0$ or $x-3=0$

$2x=-1$ $\quad$ $x=3$

$x=-\dfrac{1}{2}$

Check:

$x = -\frac{1}{2}$

LS	RS
$\frac{1}{\sqrt{x+1}}$	$\frac{\sqrt{2x+3}}{2x}$
$\frac{1}{\sqrt{-\frac{1}{2}+1}}$	$\frac{\sqrt{2\left(-\frac{1}{2}\right)+3}}{2\left(-\frac{1}{2}\right)}$
$\frac{1}{\sqrt{\frac{1}{2}}}$	$\frac{\sqrt{-1+3}}{-1}$
$\frac{1}{\sqrt{\frac{1}{2}}}$	$\frac{\sqrt{2}}{-1}$
$\frac{1}{\sqrt{\frac{1}{2}}}$	$\frac{-\sqrt{2}}{1}$

LS $\neq$ RS
(does not check)

$x = 3$

LS	RS
$\frac{1}{\sqrt{x+1}}$	$\frac{\sqrt{2x+3}}{2x}$
$\frac{1}{\sqrt{3+1}}$	$\frac{\sqrt{2(3)+3}}{2(3)}$
$\frac{1}{\sqrt{4}}$	$\frac{\sqrt{6+3}}{6}$
$\frac{1}{2}$	$\frac{\sqrt{9}}{6}$
$\frac{1}{2}$	$\frac{3}{6}$
$\frac{1}{2}$	$\frac{1}{2}$

LS = RS
(checks)

The solution is 3. The root $x = -\frac{1}{2}$ is extraneous and must be rejected.

2. Let x be the required number.

$$
\begin{aligned}
\sqrt{3x+1}+\sqrt{x-4} &= 5\\
\sqrt{3x+1} &= 5-\sqrt{x-4}\\
\left(\sqrt{3x+1}\right)^2 &= \left(5-\sqrt{x-4}\right)^2\\
3x+1 &= 25-10\sqrt{x-4}+x-4\\
3x+1 &= 21+x-10\sqrt{x-4}\\
2x-20 &= -10\sqrt{x-4}\\
(2x-20)^2 &= \left(-10\sqrt{x-4}\right)^2\\
4x^2-80x+400 &= 100(x-4)\\
4x^2-80x+400 &= 100x-400\\
4x^2-180x+800 &= 0\\
4\left(x^2-45x+200\right) &= 0\\
x^2-45x+200 &= 0\\
(x-40)(x-5) &= 0
\end{aligned}
$$

$x-40=0$ or $x-5=0$

$x=40 \qquad x=5$

Check:

$x=40$

LS	RS
$\sqrt{3x+1}+\sqrt{x-4}$	5
$\sqrt{3(40)+1}+\sqrt{40-4}$	5
$\sqrt{120+1}+\sqrt{36}$	5
$\sqrt{121}+\sqrt{36}$	5
$11+6$	5
17	5

LS $\neq$ RS
(does not check)

$x=5$

LS	RS
$\sqrt{3x+1}+\sqrt{x-4}$	5
$\sqrt{3(5)+1}+\sqrt{5-4}$	5
$\sqrt{15+1}+\sqrt{1}$	5
$\sqrt{16}+\sqrt{1}$	5
$4+1$	5
5	5

LS = RS
(checks)

The required number is 5. The root 40 is extraneous and must be rejected.

Mathematics 33
9MA33P21
L.R.D.C.
Producer
SECOND EDITION
1991